Asset Pricing and Ambiguity: Empirical Evidence*

Menachem Brenner† and Yehuda Izhakian‡

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Abstract

This paper introduces ambiguity in conjunction with risk to study the risk–ambiguity–return relationship. It develops empirical methodologies for measuring the *degree of ambiguity* and for assessing *attitudes* toward ambiguity. The main finding is that ambiguity in the stock market is priced—a possible explanation for the *equity premium puzzle*. Eliciting preferences for ambiguity, it also finds that investors' level of aversion to or love for ambiguity is contingent upon the expected probability of favorable returns. The empirical methodology of measuring ambiguity could be useful for introducing ambiguity into other economic and financial studies.

Keywords and Phrases: Ambiguity aversion, Ambiguity measurement, Knightian uncertainty, Equity premium.

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†Stern School of Business, New York University; mbrenner@stern.nyu.edu

‡Stern School of Business, New York University; and Zicklin School of Business, Baruch College; yud@stern.nyu.edu

1 Introduction

Referring to the equity premium puzzle, Lucas (2003) states that "*It would be good to have the equity premium resolved, but I think we need to look beyond high levels of risk aversion to do it.*"[1] The current paper proposes looking at a dimension of uncertainty orthogonal to risk—the uncertainty of probabilities that make up risk, so-called *ambiguity* or *Knightian uncertainty*. In equity markets, risk refers to conditions where future returns to be realized are unknown with given probabilities, while ambiguity refers to the situation where the probabilities associated with these possible realizations are unknown or not uniquely assigned. An obvious question to ask is, therefore, whether ambiguity can provide a resolution to the equity premium puzzle.

This paper investigates the risk–ambiguity–return relationship in the equity markets. To this end, it develops a new empirical methodology for measuring the degree of ambiguity using market data. With this measure in hand, the paper shows that, when introducing ambiguity in conjunction with risk, risk has a significant positive effect on the expected return, which is otherwise found to be negative or insignificant. Ambiguity itself is found to be an important pricing factor. Thereby, the paper provides a possible explanation for the *equity premium puzzle*, suggesting that ambiguity is a missing factor.

The effect of ambiguity on the equity premium—the *ambiguity premium*—is found to be contingent upon the expected probability of favorable returns and upon the investors' related preferences. Introducing the idea of *probabilistic contingent preferences* concerning ambiguity, the paper elicits investors' attitude toward ambiguity and finds that their aversion to ambiguity increases with the expected probability of favorable returns (gain), and that their love for ambiguity increases with the expected probability of unfavorable returns (loss). The ambiguity premium is determined accordingly: love for ambiguity results in a negative premium while aversion to ambiguity results in a positive premium.

Empirical asset pricing studies, until recently, have typically ignored ambiguity and have not dealt with its potential effect on prices. As early as Merton (1980), numerous studies have investigated the fundamental (linear) relationship between the risk and return of the market portfolio in the mean–variance paradigm. The findings have been conflicting, ranging from a positive relationship (e.g., French *et al.* (1987), Campbell and Hentschel (1992), and Goyal and Santa-Clara (2003)) to a negative one (e.g., Campbell (1987) and Nelson (1991)). Over the years, many studies have attempted

[1] Barillas *et al.* (2009) open their paper with this quotation to motivate their attempt to reinterpret the risk aversion parameter as containing an element of aversion to model uncertainty. Epstein (2010) also uses this quotation to express the idea that fitting moments is not sufficient for supporting a theoretical model.

to reconcile the conflicting findings. Some have dealt with the econometric methodology employed to estimate the conditional variance (e.g., Glosten *et al.* (1993) and Harvey (2001)) while others have proposed alternative risk measures (e.g., Ghysels *et al.* (2005)). Another strand of the literature has introduced time-varying elements such as regime switching (e.g., Campbell and Cochrane (1999), and Whitelaw (2000)), time-varying risk aversion (e.g., Campbell and Cochrane (1999)) and investor sentiment (e.g., Yu and Yuan (2011)). One of the most comprehensive empirical investigations of the risk–return relationship is Welch and Goyal (2008), which finds the variables suggested in previous studies to be poor predictors of the equity premium.

We suspect that the mixed findings about the risk–return relationship are due to the embedding of ambiguity in risk, which conceals the potentially separate effects of "net risk" and ambiguity. Recent theoretical studies have introduced ambiguity into asset pricing theory and suggested that a missing factor, determining the expected return, is ambiguity (e.g., Epstein and Schneider (2010), Ui (2011), and Izhakian and Benninga (2011)). Along the same lines, there has been a surge in research that focuses on other theoretical aspects of ambiguity, such as limited market participation (e.g., Dow and Werlang (1992)), portfolio inertia (e.g., Illeditsch (2011)) and extensions of the CAPM that incorporate ambiguity (e.g., Chen and Epstein (2002) and Izhakian (2012)). Some papers support their models by calibrating them to the data (e.g., Epstein and Schneider (2008) and Ju and Miao (2012)) while others use proxies for ambiguity, such as disagreement among analysts (e.g., Anderson *et al.* (2009) and Antoniou *et al.*. (2014)). Most empirical (behavioral) studies about ambiguity use data collected in controlled experiments and focus on individuals' attitudes toward ambiguity rather than on the implications of ambiguity itself. Only a few studies use market data to proxy for ambiguity; for example, Williams (2014) uses VIX and Ulrich (2013) uses entropy of inflation.

Though there is an abundance of research on various aspects of ambiguity and aversion to it, we are not aware of any empirical study that develops a theoretically based measure of ambiguity using market data directly, and that introduces this measure into the risk–return relationship. A possible explanation for this lack might be that only recently has there been a theoretical development that has attained a separation between risk and ambiguity and between tastes and beliefs in a way that enables the empirical measurement of ambiguity. This development has been attained by the theoretical framework proposed by Izhakian (2014a, 2014b) which provides the basis for the current paper. This framework proposes a *measure of ambiguity*, which is independent of risk and of attitudes toward risk and ambiguity. Namely, it proposes that the degree of ambiguity, denoted $\mho^2$, can be measured by the expected volatility of probabilities, across the relevant events.[2] The intuition of $\mho^2$ is that, as the degree of risk may be measured by the volatility of returns, so too can the degree of ambiguity be measured by the volatility of the probabilities of returns. This measure of ambiguity can also simply be viewed as a nonlinear aggregate of the variance of the mean, the variance of the variance and the variances of all other probability moments, through the variance of the probability density function. This stake-independent measure $\mho^2$ is a centerpiece of the empirical tests employed in the current paper.

Our point of departure is the underlying hypothesis that ambiguity is one of the determinants of the equity premium. This hypothesis relies upon Izhakian (2014b), which suggests that the risk–ambiguity–return relationship can be defined by

$$\mathbb{E}_t\left[r_{t+1}\right] - r_f \;=\; \gamma\frac{1}{2}\mathbb{V}\mathrm{ar}_t\left[r_{t+1}\right] + \eta\big(1 - \mathrm{E}_t[\mathrm{P}_{t+1}]\big)\mathbb{E}_t\big[\left|r_{t+1} - \mathbb{E}_t\left[r_{t+1}\right]\right|\big]\mho_t^2\left[r_{t+1}\right], \tag{1}$$

where r_f is the risk-free rate of return, r is the return on the market portfolio, $\mathbb{V}\mathrm{ar}_t\left[r_{t+1}\right]$ is the degree of risk, measured by the volatility of the market return, γ measures the aversion to risk of a representative investor or an aggregation of the risk aversion coefficients of investors, $\mho_t^2\left[r_{t+1}\right]$ measures the degree of ambiguity, and η measures the representative investor's attitude toward ambiguity, which is contingent upon the expected probability of an unfavorable return, $\mathrm{E}_t\left[\mathrm{P}_{t+1}\right]$.[3] The expectation, the variance, and the ambiguity of the market return are conditional upon the information available at the beginning of the return period, time t.

The equity premium (uncertainty premium), defined in Equation (1), has two separate components: a risk premium and an ambiguity premium. To see the intuition for the latter premium, recall that the conventional *risk premium* can be viewed as the premium that an investor is willing to pay for exchanging a risky bet for a riskless one with an identical expected outcome. In a similar way, the *ambiguity premium* can be viewed as the premium that an investor is willing to pay for exchanging a risky-ambiguous bet for a risky but non-ambiguous bet with an identical expected outcome and identical risk.

We test four hypotheses implied by the theoretical asset pricing model. The first hypothesis is that, when ambiguity is insulated from risk, risk will have a positive and significant effect on the equity premium, i.e., there will be a positive risk premium. The second hypothesis is that, for a high expected probability of *unfavorable* returns, the ambiguity premium will be negative, implying

[2] Risk is orthogonal to ambiguity as suggested by Izhakian (2014b) and verified empirically in the current paper by the low and insignificant correlation between these two factors.

[3] For example, a return lower than the risk-free rate can be considered unfavorable.

ambiguity-loving behavior. The third hypothesis is that, for a high expected probability of *favorable* returns, the ambiguity premium will be positive, implying *ambiguity-averse* behavior. The fourth hypothesis is that aversion to ambiguity increases with the expected probability of favorable returns, and love for ambiguity increases with the expected probability of unfavorable returns. This implies that the magnitude of the ambiguity premium increases with the expected probabilities.

To test these hypotheses empirically, the scope of the general model proposed in Equation (1) is narrowed based upon previous findings of the experimental literature that has found that individuals typically exhibit constant relative risk aversion (e.g., Schechter (2007) and Cohen and Einav (2007)). The experimental literature has also found that typically individuals have asymmetric ambiguity preferences for unfavorable and favorable outcomes (e.g., Abdellaoui *et al.* (2005), and Du and Budescu (2005)), but it has not yet identified the functional form of attitude toward ambiguity. Accordingly, the empirical tests of our hypotheses are designed without imposing a particular functional structure on attitudes toward ambiguity. As the model in Equation (1) dictates, the test do account for ambiguity preferences, which are contingent upon the expected probability of favorable returns (gains) in time-series regressions of the risk–ambiguity–return relationship. With this design in place, we explore the (nonlinear) relationship between ambiguity and the market expected return in a time-series context. To conduct these investigations, we use the exchange traded fund (ETF), SPDR, on the S&P500 index as a proxy for the market portfolio.

The empirical findings support our hypotheses, showing that ambiguity significantly affects stock market returns. That is to say, investors take into account the degree of ambiguity when they price financial assets. The findings provide strong evidence that individuals are ambiguity averse when it comes to favorable returns and ambiguity loving when it comes to unfavorable returns. Moreover, their aversion to ambiguity increases with the expected probability of favorable returns and their love for ambiguity increases with the expected probability of unfavorable returns. Introducing ambiguity separately from risk (volatility) shows that the expected volatility of the market portfolio has a positive and significant effect on its expected return. These findings provide a possible explanation for previous puzzling results about the risk–return relationship.

Our findings further the understanding of the nature of attitudes toward ambiguity. The behavioral literature documents that investors who face a high probability of losses typically tend to embrace ambiguity, while if they face a high probability of gains they may exhibit aversion to ambiguity. For example, Viscusi and Chesson (1999) find that people exhibit aversion to ambiguity ("fear" effects) for small probabilities of loss and love for ambiguity ("hope" effects) for large probabilities of loss.

Assuming risk neutrality, Maffioletti and Michele (2005) find ambiguity seeking in individuals' trading behavior. Statistically analyzing information regarding health insurance, Wakker *et al.* (2007) document that individuals are ambiguity seeking. Other behavioral studies that find ambiguity-loving behavior when there is a relatively high probability of loss and ambiguity aversion when there is a relatively high probability of gain include Mangelsdorff and Weber (1994), Abdellaoui *et al.* (2005), and Du and Budescu (2005). Our empirical findings are consistent with these studies, showing that investors have asymmetric preferences for ambiguity. Moreover, these findings enable an analysis of aggregate preferences concerning ambiguity and contribute to the previous literature by allowing for the identification of the particular functional form of attitudes toward ambiguity.

To verify that our findings are driven by ambiguity and related preferences and not by other potential risk factors, we conduct an exhaustive series of robustness tests. Among other tests, we control for skewness, kurtosis, volatility of the mean and volatility of volatility. In addition, we test for the effect of investors' sentiment (Baker and Wurgler (2006)) and downside risk (Ang *et al.* (2006)) alongside ambiguity. In all these tests the effect and significance of ambiguity remain, while the other factors are mostly insignificant. We also conduct a series of tests to verify that our results are derived from a set of probability distributions and could not have been derived from a (possible) single distribution.

The rest of the paper is organized as follows. Section 2 provides the theoretical framework. Section 3 discusses the data and develops the estimation methodology. Section 4 designs the regression tests and presents the empirical findings. Section 5 tests for robustness. Section 6 provides a summary and conclusions. The appendix discusses alternative ambiguity measures.

2 The theoretical model

The expected utility paradigm has been dominating modern asset pricing theories since their inception in the late 1950s. The main underlying assumption of this paradigm is that investors know, or act as if they know, the probabilities of future returns.[4] As Ellsberg (1961) demonstrates, a fundamental issue with this paradigm is that, when probabilities are not precisely known, many decisions cannot be justified by classical expected utility theory, suggesting that ambiguity—the uncertainty about probabilities (Knightian uncertainty)—may play an important role in decision processes and consequently in asset pricing.

[4]Expected utility theory, introduced by von Neumann-Morgenstern (1944), assumes that probabilities are *objectively* given, while Savage (1954) suggests that probabilities are *subjectively* determined.

Knightian uncertainty has provided the basis for a rich body of literature in decision theory, stimulated by the Ellsberg (1961) paradox. The resulting models include the max-min expected utility with multiple priors (MEU) of Gilboa and Schmeidler (1989), the subjective nonadditive probabilities of Gilboa (1987), the Choquet expected utility of Schmeidler (1989), the cumulative prospect theory of Tversky and Kahneman (1992), the α-MEU of Ghirardato *et al.*. (2004), the smooth model of ambiguity of Klibanoff *et al.* (2005), the model misspecification of Hansen and Sargent (2001) and others. These models have tried to satisfy the need for an applicable framework that can be used in empirical investigations. This requires the separation of ambiguity from risk and of tastes from beliefs in such a way that ambiguity may be measured and incorporated into the model.

Recently, Izhakian (2014a) has introduced a decision-making model, called *expected utility with uncertain probabilities* (henceforth EUUP), that allows for a stake-independent measurement of the degree of ambiguity. The main idea of EUUP is that, in the presence of ambiguity—when probabilities are themselves uncertain—preferences concerning ambiguity are applied to these probabilities such that aversion to ambiguity is defined as aversion to *mean-preserving spreads in probabilities*. As such, the Rothschild and Stiglitz (1970) approach, which is typically applied to outcomes when measuring risk, can also be applied to probabilities when measuring ambiguity. Unlike other measures of ambiguity that are stake-dependent and consider only the variance of a single moment of the distribution, i.e., the variance of the mean or the variance of the variance, the resulting measure is stake-independent and accounts for all the moments of the return distribution.[5]

Consider a decision to save one unit of wealth. The future consumption, determined by the one-period (uncertain) return r, is $1 + r$. To define the uncertain return formally, let $(\Omega, \mathcal{F}, \mathrm{P})$ be a probability space, where $\mathrm{P} \in \mathcal{P}$ is a probability measure, and the set of probability measures $\mathcal{P}$ is closed and convex. The σ-algebra Π of subsets of $\mathcal{P}$ is equipped with a Borel probability measure, denoted χ, with a bounded support. An uncertain return is then defined by the variable $r : \Omega \to \mathbb{R}$. EUUP proposes that the expected utility associated with the decision above is

$$\begin{aligned} \mathrm{V}(c) \;=\; & \int_{-\infty}^{0} \left[\Gamma^{-1}\left(\int_{\mathcal{P}} \Gamma\big(\mathrm{P}\left(\mathrm{U}\left(1+r\right) \geq z\right)\big) d\chi \right) - 1 \right] dz \, + \\ & \int_{0}^{\infty} \Gamma^{-1}\left(\int_{\mathcal{P}} \Gamma\big(\mathrm{P}\left(\mathrm{U}\left(1+r\right) \geq z\right)\big) d\chi \right) dz, \end{aligned} \tag{2}$$

where $\mathrm{U} : \mathbb{R} \to \mathbb{R}$ is a (von Neumann-Morgenstern) *utility function*, capturing attitude toward risk,

[5] Other models (e.g., Gilboa and Schmeidler (1989) and Klibanoff *et al.* (2005)) do not allow for such a derivation. For example, in Klibanoff *et al.* (2005) aversion to ambiguity is defined as aversion to mean-preserving spreads in certainty-equivalent utilities, which are subject to risk and preferences for risk. For this reason, measuring ambiguity is limited within other frameworks.

$\Gamma : [0,1] \to \mathbb{R}$ is an *outlook function*, capturing attitude toward ambiguity, and $\mathrm{P}(\cdot)$ is the cumulative probability under $\mathrm{P} \in \mathcal{P}$.[6] In particular, as with risk attitudes, a concave Γ characterizes ambiguity-averse behavior, a convex Γ characterizes ambiguity-loving behavior, and a linear Γ characterizes ambiguity-neutral behavior.[7] The utility function is assumed to satisfy $\mathrm{U}(1 + r_f) = 0$, where the risk-free rate r_f is the *reference point* relative to which returns are classified as unfavorable (losses) or as favorable (gains).[8] That is, any return lower than r_f is considered unfavorable and any return higher than r_f is considered favorable.

To extract the ambiguity premium, assume that both U and Γ are strictly-increasing and twice-differentiable functions. Then, by Izhakian (2014b), the uncertainty premium of a risky and ambiguous consumption c in Equation (2) is[9]

$$\mathcal{K} \approx \underbrace{-\frac{1}{2}\frac{\mathrm{U}''(1+\mathrm{E}[r])}{\mathrm{U}'(1+\mathrm{E}[r])}\mathbb{V}\mathrm{ar}[r]}_{\mathcal{R}}\underbrace{-\mathbb{E}\left[\frac{\Gamma''(1-\mathrm{E}[\mathrm{P}(r)])}{\Gamma'(1-\mathrm{E}[\mathrm{P}(r)])}\right]\mathbb{E}\big[\,|r-\mathbb{E}[r]|\,\big]\mho^2[r]}_{\mathcal{A}}. \tag{3}$$

This premium is required by investors in return for bearing the risk and ambiguity associated with holding the asset. It is separated into two components. The first, $\mathcal{R}$, is the risk premium and the second, $\mathcal{A}$, is the ambiguity premium.[10] The expected return $\mathbb{E}[r]$ and the variance of return $\mathbb{V}\mathrm{ar}[r]$ are computed using expected probabilities, i.e., double expectations with respect to uncertain returns and with respect to their uncertain probabilities.

The *measure of ambiguity* is defined by

$$\mho^2[r] = \int \mathrm{E}[\varphi(r)]\,\mathrm{Var}[\varphi(r)]\,dr,$$

where $\varphi(\cdot)$ is an uncertain probability density function, and the expectation $\mathrm{E}[\cdot]$ and the variance $\mathrm{Var}[\cdot]$ are taken with respect to second-order probabilities (probabilities over the set of probability distributions) χ. The main idea of $\mho^2$ is that ambiguity can be measured by the volatility of probabilities, just as the degree of risk can be measured by the volatility of returns (Rothschild and Stiglitz, 1970). In general, irrespective of the decision-theoretic framework, $\mho^2$ encompasses not only

[6] The function V puts more structure on the functional representation proposed by Wakker (2010) and Kothiyal *et al.* (2011) for Tversky and Kahneman's (1992) *cumulative prospect theory*. It applies a two-sided Choquet integration (Schmeidler, 1989) to favorable returns and to unfavorable returns, relative to $1 + r_f$. Unlike cumulative prospect theory, it assumes neither different attitudes toward risk for losses and for gains nor loss aversion.

[7] Notice that, when investors are ambiguity neutral, Equation (2) reduces to the conventional expected utility.

[8] Previous literature focuses on the implications of losses and gains for preferences (e.g., Barberis and Huang (2001) and Hirshleifer (2001)), while EUUP focuses on beliefs.

[9] To prove this approximation, it is assumed that returns and their probabilities are close to their expectation. The proof applies the same method as is used in Arrow (1965) and Pratt (1964), first to probabilities and then to returns.

[10] Izhakian and Benninga (2011) and Ui (2011), for example, have also derived an ambiguity premium. In their models the ambiguity premium is also a function of risk attitudes, whereas in our model the ambiguity premium is solely a function of ambiguity and the attitudes toward it.

an ambiguous variance and an ambiguous mean but also the ambiguity of the higher moments of the probability distribution (i.e., skewness, kurtosis, etc.) through the uncertainty of probabilities. This stake-independent measure, $\mho^2 \in [0, \infty)$, attains its minimal value, 0, only when the probabilities are known.

To see the intuition behind the concept of ambiguity in EUUP, consider the following binomial example of an asset with two possible future returns: $d = -10\%$ and $u = 20\%$. Assume that the probabilities of d and u are known, say $\mathrm{P}(d) = \mathrm{P}(u) = 0.5$. The expected return is thus 5%, and the standard deviation of the return (measuring the degree of risk) is 15%. In this case, since the probabilities are precisely known, ambiguity is not present ($\mho = 0$) and investors only face risk. Assume now that the probabilities of d and u can be either $\mathrm{P}(d) = 0.4$ and $\mathrm{P}(u) = 0.6$ or alternatively $\mathrm{P}(d) = 0.6$ and $\mathrm{P}(u) = 0.4$, where these two alternative distributions are equally likely. Investors now face not only risk but also ambiguity. The degree of ambiguity, in terms of the probabilities, is $\mho = 0.1$. Notice that the degree of risk, computed using the expected probabilities $\mathrm{E}[\mathrm{P}_d] = \mathrm{E}[\mathrm{P}_u] = 0.5$, has not changed.

The ambiguity measure $\mho^2$ is a centerpiece of our empirical tests. To estimate the degree of ambiguity from market data, the probabilities of returns must be derived first. To do so, it is assumed that there is a representative investor, whose set of priors is an aggregation of the sets of priors of all investors in the economy, and who acts as if all priors within this set are equally likely.[11] It is also assumed that each subset of observed returns on the market is the result of a realization of one prior out of this set of priors. In particular, every trading day is characterized by a different distribution of returns P, and the set $\mathcal{P}$ of these distributions over a month represents the investor's set of priors. In addition, returns on the market portfolio are assumed to be normally distributed but not i.i.d. That is, every $\mathrm{P} \in \mathcal{P}$ is normal, governed by a different mean μ and variance σ^2.[12] The degree of ambiguity in our setting is, therefore, measured by

$$\mho^2[r] = \int \mathrm{E}[\phi(r;\mu,\sigma)] \, \mathrm{Var}[\phi(r;\mu,\sigma)] \, dr, \tag{4}$$

where ϕ stands for the normal probability density function.

Based upon findings reported in behavioral studies, it is assumed that the representative inventor's attitude toward risk is of the constant relative risk aversion (CRRA) class.[13] However, since the

[11] A representative investor can be defined as one whose tastes and beliefs are such that, if all investors in the economy had identical tastes and beliefs, the equilibrium in the economy would remain unchanged; see, for example, Constantinides (1982).

[12] Each, the volatility of μ and volatility of σ^2, has separately been attributed to ambiguity (e.g., Maccheroni *et al.* (2013) and Faria and Correia-da-Silva (2014)).

[13] See, for example, Kachelmeier and Shehata (1992), Chetty (2006), Schechter (2007) and Cohen and Einav (2007).

literature has not provided conclusive evidence regarding the nature of attitudes toward ambiguity, no particular functional structure is imposed on Γ.[14] The uncertainty premium, defined by Equation (3), can thus be simplified to

$$\mathcal{K} \approx \gamma \frac{1}{2} \mathbb{V}\mathrm{ar}[r] + \mathbb{E}\left[\eta\left(1 - \mathrm{E}\left[\mathrm{P}(r)\right]\right)\right] \mathbb{E}\left[\,|r - \mathbb{E}[r]|\,\right] \mho^2[r], \tag{5}$$

where γ is the coefficient of relative risk aversion, and $\eta(\cdot) = -\frac{\Gamma''(\cdot)}{\Gamma'(\cdot)}$ characterizes the attitude toward ambiguity conditional upon the expected cumulative probability $\mathrm{P}(r) = \Phi(r; \mu, \sigma)$, where Φ stands for the normal cumulative probability distribution. A positive (negative) γ implies aversion to risk (love of risk), and a positive (negative) $\eta(\cdot)$ implies aversion to ambiguity (love of ambiguity). The empirical tests that follow are designed to allow attitude toward ambiguity, formed by $\eta(\cdot)$, to be different for favorable and unfavorable returns, as well as for different levels of $\mathrm{E}[\mathrm{P}(r)]$.

The effect of *uncertainty* on returns is now represented by two terms: a risk term and an ambiguity term. In other words, Equation (5) divides the *equity premium* into two distinct premiums: the risk premium and the ambiguity premium. Each is measured separately and may have a different effect on excess returns.

We next present our hypotheses. Our first hypothesis is standard in the asset pricing literature.

Hypothesis 1 *The risk premium should be positive since typically investors exhibit risk aversion.*

Our next two hypotheses are related to individuals' preference for ambiguity as reflected in Equation (5). They are based upon earlier behavioral findings regarding individuals' preferences for ambiguity, which are shown to be different for losses and for gains; see, for example, Maffioletti and Michele (2005), Abdellaoui *et al.* (2005) and Wakker, *et al.* (2007).

Hypothesis 2 *Investors typically exhibit love for ambiguity when expecting unfavorable returns. Therefore, for a relatively high expected probability of unfavorable returns, the ambiguity premium should be negative.*

Hypothesis 3 *Investors typically exhibit aversion to ambiguity when expecting favorable returns. Therefore, for a relatively high expected probability of favorable returns, the ambiguity premium should be positive.*

Our fourth hypothesis extends further the discussion of preferences for ambiguity contingent upon the expected probability of unfavorable returns (losses) and favorable returns (gains) and their con-

[14] In particular, the question of whether individuals are typified by a constant relative ambiguity aversion, constant absolute ambiguity aversion or perhaps a decreasing (increasing) relative (absolute) ambiguity aversion has not yet been settled.

sequential magnitude of the ambiguity premium. This hypothesis is based upon earlier behavioral findings of Mangelsdorff and Weber (1994), Viscusi and Chesson (1999), and Du and Budescu (2005).

Hypothesis 4 *Aversion to ambiguity increases with the expected probability of favorable returns and love for ambiguity increases with the expected probability of unfavorable returns. Therefore, the higher is the expected probability of favorable returns, the higher is the positive ambiguity premium. On the other hand, the higher is the expected probability of unfavorable returns, the higher is the negative ambiguity premium.*

In the following sections we present the empirical tests of the model in Equation (5) and the related hypotheses. We first provide a new methodology for measuring the variables, especially ambiguity, and then test the model empirically.

3 Data and ambiguity measurement

3.1 Data

The main body of data used in our empirical research is intraday data on the ETF SPDR taken from the TAQ database.[15] The Standard & Poor's Depositary Receipts (SPDR) is designed to track the Standard & Poor's 500 (S&P500) index, which is considered to represent the equity market in the U.S. The stocks in the SPDR have the same weights as in the index, its expense ratio is about 7-8 basis points, and its bid-ask spread is 1-2 basis points. The quarterly dividends are added to the SPDR every three months and it can be sold short like any other stock. A typical volume for the SPDR is between 100 and 150 million shares per day, which is the highest of any US stock traded on any exchange.

We use the SPDR as a proxy for the market portfolio and not the S&P500 index itself since the SPDR trades continuously, while the index contains illiquid stocks so its values may be stale. The data cover the period from February 1993 to December 2013, 251 months in total.[16] Daily and monthly returns, adjusted for dividends, are obtained from the CRSP database. VIX values are obtained from the CBOE site. The risk-free rate is the one-month Treasury bill rate of return, provided by Ibbotson Associates.

[15]The Trade And Quotes (TAQ) database; Wharton Research Data Services (WRDS).

[16]Under the ticker symbol SPY, the SPDR began trading on the American Stock Exchange (AMEX) on January 29, 1993.

3.2 Computing ambiguity

The first step in designing the empirical tests is to develop a method to compute the time-series values of the monthly degree of ambiguity.[17] We take the prices of the SPDR every five minutes from 9:30 to 16:00 each day, giving 79 prices in total for each day.[18] If there was no trade at a specific time, we take the volume-weighted average of the closest trading prices within five minutes of that time stamp. By not including returns between the closing prices and the opening prices of the following day, we eliminate the impact of overnight price changes (new information) and dividend distributions. Using these prices, we compute the five-minute returns, giving a maximum of 78 returns in total for each day. Observations with extreme price changes (minus or plus 10% log returns) within five minutes are omitted. These observations are dropped because many of them are probably due to erroneous orders that were cancelled by the stock exchange.[19]

The choice of five-minute intervals is dictated by the measure of ambiguity. To perform meaningful time-series tests, in our 21-year period (February 1993 to December 2013), we need to use monthly observations. To obtain a statistically meaningful monthly measure of ambiguity, a daily estimate of a probability distribution is needed. This, in turn, requires intraday observations. The SPDR is selected since it is frequently traded and its bid-ask spread is minimal, such that the bias in return variances is minimal. The decision to compute returns using five-minute intervals is motivated by Andersen *et al.* (2001), who show that this time interval is sufficient to minimize microstructure effects.

For each day there are between 33 and 78 observations. We use these observations to compute the normalized (by the number of intraday observations) daily mean and variance of the return, denoted μ and σ^2 respectively. As in French *et al.* (1987), the variance of the returns is computed by applying the adjustment for non-synchronous trading, proposed by Scholes and Williams (1977).[20] Based upon the assumption that the intraday returns are normally distributed, we construct the set of priors $\mathcal{P}$, where each prior P within the set $\mathcal{P}$ is defined by a pair of μ and σ. It is important to mention that, in our approach, the set of priors $\mathcal{P}$ that underlies ambiguity is endogenously derived. To the best of our knowledge, all other empirical and experimental studies except for Hey *et al.* (2010) take the set

[17] We focus on one-month intervals; however, the same procedure can be applied to periods of less (or longer) than one month, 10 trading days for example.

[18] We also test our model using 10-minute time-intervals; the results are essentially the same. To check for robustness, we also perform our tests using only the prices from 10:00 to 15:30, i.e., eliminating the impact of the trading noise caused by opening and closing daily positions during the first and last half-hour of the trading day; see, for example, Lockwood and Linn (1990) and Heston *et al.* (2010). Again, the results remain unchanged.

[19] Testing the model including these observations shows that the effect of ambiguity is even more significant than when we exclude these observations.

[20] Scholes and Williams' (1977) adjustment for non-synchronous trading suggests that the volatility of returns takes the form $\sigma_t^2 = \sum_{i=1}^{N_t} \left(r_{t,i} - \mathrm{E}\left[r_{t,i}\right]\right)^2 + 2\sum_{i=2}^{N_t} \left(r_{t,i} - \mathrm{E}\left[r_{t,i}\right]\right)\left(r_{t,i-1} - \mathrm{E}\left[r_{t,i-1}\right]\right)$. We also test our model without the Scholes-Williams correction for non-synchronous trading. The results are essentially the same.

of priors to be exogenously given or designed by the experimenter.

Given the set P of (normal) probability distributions, we first compute for each day (prior) the probability of unfavorable returns (loss), $\mathrm{P}(r < r_f) = \Phi(r_f; \mu, \sigma)$, where any return lower than the risk-free rate is considered unfavorable.[21] For each month, there are 20 to 22 different loss probabilities. Their expectation, $\mathrm{E}[\mathrm{P}(r < r_f)]$, is computed assuming that the daily ratios of the sample mean and the sample standard deviation, $\frac{\mu}{\sigma}$, are Student's-t distributed, which implies that the cumulative probabilities of unfavorable returns, $\mathrm{P}(r < r_f)$, are uniformly distributed across the month.[22] This method assigns lower weights to values of $\frac{\mu}{\sigma}$ that deviate from the monthly mean of $\frac{\mu}{\sigma}$. The expected probabilities of unfavorable returns will be used in estimating attitude toward ambiguity.

To compute the monthly degree of ambiguity, specified in Equation (4), we represent each daily return distribution by a histogram. To this end, we divide the range of daily returns, from -5% to 5%, into 50 intervals (bins), each of width 0.2%. For each day, we compute the probability of the return being in each bin. In addition, we compute the probability of the return being lower than -5% and the probability of the return being higher than 5%. Using these probabilities, we compute the mean and the variance of probabilities for each of the 52 bins separately. Then, we estimate the degree of ambiguity of each month by the discrete form

$$\mho^2[r] = \frac{1}{w \ln \frac{1}{w}} \begin{pmatrix} \mathrm{E}\big[\Phi(r_0; \mu, \sigma)\big]\mathrm{Var}\big[\Phi(r_0; \mu, \sigma)\big] + \\ \sum_{i=1}^{50} \mathrm{E}\big[\Phi(r_i; \mu, \sigma) - \Phi(r_{i-1}; \mu, \sigma)\big]\mathrm{Var}\big[\Phi(r_i; \mu, \sigma) - \Phi(r_{i-1}; \mu, \sigma)\big] + \\ \mathrm{E}\big[1 - \Phi(r_{50}; \mu, \sigma)\big]\mathrm{Var}\big[1 - \Phi(r_{50}; \mu, \sigma)\big] \end{pmatrix},$$

where $r_0 = -0.05$, $w = r_i - r_{i-1} = 0.002$, and $\frac{1}{w \ln\left(\frac{1}{w}\right)}$ scales the weighted-average volatilities of probabilities to the bins' size. This scaling, which is analogous to Sheppard's correction, is tested to verify that it minimizes the effect of the selected bin size on the values of $\mho^2$. As with the expected probabilities, the variance of the probabilities is computed assuming that the daily ratios $\frac{\mu}{\sigma}$ are student's-t distributed.

It might be interesting at this point to examine the time-series of ambiguity. The upper plot in Figure 1 describes the time-series of the monthly degree of ambiguity $\mho$ over the years 1993 to 2013 and the lower plot describes the monthly excess returns on the market. Over this period we observe only a few months that contain big downward moves in the market. The two obvious ones are September

[21] We also test our model assuming that a negative return is considered unfavorable, i.e., $\mathrm{P}(r < 0)$. The results are essentially the same. In addition, we test our model while relaxing the assumption of normally distributed returns, and the results are essentially the same. Section 5 elaborates on these tests.

[22] See, for example, Kendall and Alan (2010, page 21, Proposition 1.27). This consistent with the idea that the representative investor does not have any information indicating which of the possible probability distributions is more likely, and thus he acts as if he assigns an equal weight to each possibility.

1998—the Russian default and the LTCM debacle—and September 2008, which was the time of the recent American debt crisis. It can be noticed that crisis times, typified by relatively low returns, are accompanied by relatively high levels of ambiguity. For example, in August 1998 the excess return was -13%, and a month before in July 1998 the ambiguity $\mho^2$ jumped to 0.4 and rose further to almost 0.5 in the next month. In September 2008 the excess return was -16%, while in the month before that (August 2008) ambiguity rose to 0.82 and in the following month it jumped to a level higher than 1.3. Another example is the "foot-and-mouth crisis" that began in February 2001, and was seemingly the reason for the ambiguity level jumping to 0.56 at that time.

[INSERT FIGURE 1]

The monthly risk (volatility) is measured by the variance of daily returns. This variance is also computed by applying the adjustment for non-synchronous trading, proposed by Scholes and Williams (1977). Figure 2 describes the time-series of the volatility (in terms of standard deviation) over the years 1993 to 2013.

[INSERT FIGURE 2]

3.3 Descriptive statistics

The building blocks for measuring the degree of ambiguity in the market are the daily means and variances, computed from the five-minute returns. Panel A of Table 1 provides descriptive statistics of the intraday returns. The statistics are reported in daily terms. The average number of five-minute returns is 69.79 per day. The average probability of unfavorable returns, P, is 0.492. These probabilities are computed using the ratio $\frac{\mu}{\sigma}$, which ranges between -10.1 and 9.5, with an average of 0.056. A closer look at the source of the variation in this ratio shows that it is driven by the variations in both μ and σ. Over the entire sample, the standard deviation of μ, in terms of the daily return, is 1% while the standard deviation of σ is 0.5%. It is important to note that the mean of the realized returns, measured over short intervals, is a poor proxy for the annual expected return; i.e., its standard error is very large. In our context, however, daily probabilities are extracted from the ratio of $\frac{\mu}{\sigma}$, whose distribution gives very little weight to extreme observations. The probabilities obtained appear to be very reasonable estimates. For example, on average, the probabilities of unfavorable returns should be lower than 0.5 and in our sample their average is 0.492, even though on some days the realized probability exceeds 0.5. The standard deviation of the probabilities of unfavorable returns over the entire sample is 0.3.

[INSERT TABLE 1]

Panel B of Table 1 provides descriptive statistics of the variables used in our empirical tests. The dependent variable is the monthly return on the SPDR, r, which serves as a proxy for the return on the market portfolio, minus the risk-free rate, r_f, which is the one-month T-bill rate. The monthly market return, r, computed using the opening price on the first trading day of the month and the closing price on the last trading day of the month, is adjusted for monthly dividends. On average, between 1993 and 2013, this return is 0.81% (about 9.72% on an annual basis). The standard deviation of r is about 4.3% monthly (about 14.9% annually).[23] On average, the monthly risk-free rate is 0.24% (about 2.88% annually), and the monthly excess return, $r-r_f$, is 0.57% (about 6.84% annually). The distribution of $r-r_f$ is somewhat negatively skewed. Most values, however, are to the right of the mean. The positive excess kurtosis, 1.15, indicates the distribution of returns has fat tails.

The monthly standard deviation, denoted $\sqrt{\nu}$, is computed from daily returns and reported on a monthly basis. The average, the monthly standard deviation computed from daily returns (across all 251 months) is about 5.2% (about 18.0% on an annual basis).[24] The monthly average absolute deviation of returns from the average return $\mathbb{E}\left[\,|r-\mathbb{E}[r]|\,\right]$, denoted ϑ, is also computed from daily returns and reported on a monthly basis. On average, ϑ (across all 251 months) is about 4.4% (about 15.3% on an annual basis). The average monthly mean -probability of unfavorable returns, $\overline{\mathrm{P}}$, is 0.492; , as expected, a bit lower than 0.50. The lowest monthly mean probability is 0.294 and the highest is 0.687. Over the period, between 1993 and 2013, the degree of ambiguity $\mho$ ranges between 0.158 and 1.303. The average ambiguity is 0.378;, the positive kurtosis indicates that the distribution of ambiguity has fat tails.

Panel B of Table 1 provides the cross-correlations among the main variables. As expected, the mean probability of unfavorable returns is strongly negatively and significantly correlated with the excess return. The excess return and volatility are also negatively and significantly correlated. However, the most important finding of this panel is that ambiguity and risk are *not* correlated, indicating that these two factors are orthogonal.

[23]Since in the short run there is slight mean reversion in returns, the standard deviation based on daily observations should be lower than that based on intraday observations.

[24]It should be noted that the average monthly standard deviation, 5.2%, is different than the standard deviation of the returns across the entire sample, reported earlier.

4 Empirical methodology and results

4.1 Empirical design

The fundamental hypothesis tested in this paper is that expected ambiguity, in addition to expected volatility (risk), has an effect on the expected return of the market portfolio. Based upon the model in Equation (5), our hypotheses suggest that the effect of ambiguity on returns is subject to the investor's attitude toward ambiguity, which in turn is contingent upon the expected probability of favorable returns.

To conduct the empirical tests, the expectations of the following four variables need to be estimated: the volatility (ν), the average absolute deviation of returns from the expected return (ϑ), the probability of unfavorable returns (P) and ambiguity ($\mho^2$). To select an appropriate estimation model, we first examine the autocorrelations, provided in Table 2. The first-order autocorrelation of the volatility is large, the second-order is somewhat lower, while the decay beyond the fifth order is relatively slow. This behavior is indicative of a non-stationary integrated moving average; see, for example, French *et al.* (1987). To address the issue of non-stationarity, we examine the changes in the natural logarithm of the standard deviation. These changes have a negative first-order autocorrelation, implying that the residuals have at least one lagged effect. The autocorrelation of the average absolute deviation, ϑ, is strongly positive for the first lag and decays consistently over the next seven lags, while the changes in its natural logarithm are negative, and meaningful only for the first-order autocorrelation. The autocorrelation of the monthly average probability of unfavorable returns $\overline{\mathrm{P}}$ is also non-stationary for the examined lags. Therefore, we examine the first-order autocorrelation of the changes in the natural logarithm of these probabilities, which is strongly negative.[25] Similarly, we observe that the ambiguity time series is non-stationary, and the autocorrelation of the changes in the natural logarithm of ambiguity is also strongly negative for one lag. To verify these observations, we conduct the Box-Pierce test for all four variables and in each test the hypothesis of an independent distribution is rejected.

[INSERT TABLE 2]

The estimated autocorrelations in Table 2 suggest that autoregressive moving average (ARMA) could be an appropriate model for estimating each of the variables: expected volatility, expected absolute deviation, expected probability and expected ambiguity. Using the formal link between

[25] Probabilities are bounded between 0 and 1. As expected, the negative autocorrelations of changes in their natural logarithm indicate that they follow a mean-reverting process; otherwise, the probabilities this variable would deviate from the range $[0, 1]$.

realized and conditional volatilities, as provided by Andersen *et al.* (2003), we estimate the expected volatilities by substituting the realized volatilities, ν, for the latent monthly volatilities.[26] To do so, for each month, $\widehat{\ln\sqrt{\nu_t}}$ is computed using the coefficients estimated by the time-series ARMA(p,q) model

$$\ln\sqrt{\nu_t} \quad = \quad \psi_0 + \epsilon_t + \sum_{i=1}^{p} \psi_i \cdot \ln\sqrt{\nu_{t-i}} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{t-i} \tag{6}$$

with the minimal Akaike information criterion (AIC). This time-series model uses the natural logarithm of volatility, $\ln\sqrt{\nu_t}$, to avoid negative expected volatility estimates and because $\sqrt{\nu_t}$ is skewed. The expected volatility is then calculated as

$$\nu_t^E \quad = \quad \exp\left(2\widehat{\ln\sqrt{\nu_t}} + 2\mathrm{Var}\left[u_t\right]\right),$$

where $\mathrm{Var}\left[u_t\right]$ is the minimal predicted variance of the error term. For every month t, using the data from month $t-35$ to month t, the time-series regression given by Equation (6) is estimated for each $p = 1, \ldots, 10$ and $q = 1, \ldots, 10$; in total $p \times q = 100$ combinations. The coefficients of the model that attains the minimal AIC (the highest-quality model) are then used for the estimation of the expected volatility. Similarly, we estimate the expected absolute deviation, ϑ, using its monthly realized values, to obtain ϑ_t^E.

Expected ambiguity is also estimated with ARMA(p,q), using a method similar to the one used to estimate the expected volatilities.[27] Namely, using realized ambiguity, the parameter $\widehat{\ln \mho_t}$ is estimated using the coefficients of the time-series model

$$\ln \mho_t \quad = \quad \psi_0 + \epsilon_t + \sum_{i=1}^{p} \psi_i \cdot \ln \mho_{t-i} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{t-1} \tag{7}$$

that attains the minimal AIC out of the $p \times q = 100$ combinations of coefficients obtained from this regression. The expected ambiguity is then calculated as

$$\left(\mho_t^2\right)^E \quad = \quad \exp\left(2\widehat{\ln \mho_t} + 2\mathrm{Var}\left[u_t\right]\right).$$

To estimate the expected probability of unfavorable returns, the parameter $\widehat{\ln Q_t}$, where $Q_t = \frac{\overline{\mathrm{P}}_t}{1-\overline{\mathrm{P}}_t}$, is estimated using the coefficients of

$$\ln Q_t \quad = \quad \psi_0 + \epsilon_t + \sum_{i=1}^{p} \psi_i \cdot \ln Q_{t-i} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{t-i} \tag{8}$$

[26] Hansen and Lunde (2005) also estimate the expected volatilities by substituting the realized volatilities for the latent volatilities.

[27] Andersen *et al.* (2003) provide the theoretical framework for integrating high-frequency intraday data into the measurement of daily volatility, and show that long-memory Gaussian vector autoregression for the logarithmic daily realized volatilities performs admirably. We apply the same approach to ambiguity, since it uses probabilities that are estimated from intraday data.

that attain the minimal AIC out of the $p \times q = 100$ combinations of coefficients of this regression. This model uses the natural logarithm of the transformed probability, $\ln Q = \ln\left(\frac{\overline{\mathrm{P}}}{1-\overline{\mathrm{P}}}\right)$, to avoid estimated probabilities that are negative or that are greater than 1. The expected probability is then calculated as

$$\mathrm{E}_t\left[\mathrm{P}_{t+1}\right] = \mathrm{P}_t^E = \frac{\exp\left(\widehat{\ln Q_t} + \frac{1}{2}\mathrm{Var}\left[u_t\right]\right)}{1 + \exp\left(\widehat{\ln Q_t} + \frac{1}{2}\mathrm{Var}\left[u_t\right]\right)}.$$

Panel A of Table 3 reports descriptive statistics of the estimated expectation of volatility, absolute deviation, the probability of unfavorable returns and ambiguity. Each expected variable is the fitted value obtained from the relevant time-series ARMA model described above. Comparing the statistics of the realized values of these variables, depicted in Table 1, to the statistics of their expectations, we observe that the dispersion of the expected values of each variable is less than the dispersion of its realized values. This can be observed from the fact that the differences between the minimal and maximal values, the variance, the skewness and the kurtosis of the estimated expected values are all lower than those of the realized values. The lower dispersion implies that the time series of the expected values of all four—volatility, absolute deviation, probability and ambiguity—are smoother than the related realized time series, which is consistent with the predictions of the estimation model. Panel B of Table 3 reports the cross-correlations among the expected values. Panel B reveals that the expected volatility is significantly correlated with VIX, which is also highly correlated with the expected absolute deviation.

[INSERT TABLE 3]

We now turn to developing the empirical tests of the risk–ambiguity–return relationship, using the estimated expected volatility, expected absolute deviation, expected probability and expected ambiguity. This relationship is defined in Equation (5) and expressed in four hypotheses. Recall that this relationship assumes CRRA attitudes, implying a linear risk–return relationship. As for ambiguity, it suggests that this relationship is a function of attitudes toward ambiguity, contingent upon the expected probability of favorable returns $1 - \mathrm{E}\left[\mathrm{P}\left(r\right)\right]$, and might be asymmetric across unfavorable and favorable returns. In particular, the ambiguity–return relationship is determined by the attitude toward ambiguity given by

$$\mathbb{E}\big[\eta\left(\mathrm{E}\left[1-\mathrm{P}\left(r\right)\right]\right)\big] \quad = \quad \mathbb{E}\left[\frac{\Gamma''\left(1-\mathrm{E}\left[\mathrm{P}\left(r\right)\right]\right)}{\Gamma'\left(1-\mathrm{E}\left[\mathrm{P}\left(r\right)\right]\right)}\right] = \int_{-\infty}^{\infty}\mathrm{E}\left[\varphi\left(r\right)\right]\frac{\Gamma''\left(1-\mathrm{E}\left[\mathrm{P}\left(r\right)\right]\right)}{\Gamma'\left(1-\mathrm{E}\left[\mathrm{P}\left(r\right)\right]\right)}dr.$$

Ideally, we would like to compute this expression directly from market data. However, the functional form of $\Gamma\left(\cdot\right)$ is unknown, so we are constrained in extracting it from the data. Therefore, to elicit the functional form of $\eta(\cdot)$, we consider only two subsets of returns: unfavorable and favorable. Accordingly, the empirical effect of attitude toward ambiguity on the ambiguity–return relationship takes the form of

$$\mathbb{E}\left[\eta\left(\mathrm{E}\left[1-\mathrm{P}\left(r\right)\right]\right)\right]=\mathrm{P}^{E}\eta\left(1-\mathrm{P}^{E}\right)+\left(1-\mathrm{P}^{E}\right)\eta\left(1-\mathrm{P}^{E}\right)=\eta\left(1-\mathrm{P}^{E}\right),$$

where $1-\mathrm{P}^{E}$ is the expected probability of a favorable return.

To retain the flexibility of asymmetric attitudes toward ambiguity, which may be nonlinearly contingent upon expected probabilities, as proposed by the model in Equation (5), we use the following design. The expected probabilities range from 0.38 to 0.61 (see Table 3). Winsorizing the very few outlier values provides the range $[0.42, 0.58]$ of expected probabilities. This range is divided into 10 equal intervals (bins) of 0.016, denoted by the index i. For example, $i=1$ denotes the probability bin $[0.42, 0.436]$. The very few values lower than 0.42 are indexed by the number $i=1$, and the very few values higher than 0.58 are indexed by the number $i=10$. The decision to use a 10-bin resolution and not a higher one is dictated by the number of observations, 216. To have meaningful statistical results, a minimal number of observations are required for each bin.

Next, a dummy variable, D, is constructed for each probability bin. If the expected probability of favorable returns in a given month t falls into bin i then the dummy variable $D_{i,t}$ is assigned the number one; otherwise it is assigned the number zero. The empirical model is then given by

$$r_t-r_{f,t} \quad = \quad \alpha+\gamma\cdot\nu_t^E+\eta\cdot\left(\left(\mho_t^2\right)^E\times\vartheta_t^E\right)+\sum_{i=2}^{10}\eta_i\cdot\left(D_{i,t}\times\left(\mho_t^2\right)^E\times\vartheta_t^E\right)+\epsilon_t. \tag{9}$$

It is important to note that, unlike attitude toward risk which is constant, attitude toward ambiguity may vary with the expected probability of favorable return. We refer to the model in Equation (9) as the *discrete model* in the sense that attitudes toward ambiguity may have finite number of degrees. We also examine a *continuous* version of the model, given by

$$r_t-r_{f,t} \quad = \quad \alpha+\gamma\cdot\nu_t^E+\eta\cdot\left(\left(\mho_t^2\right)^E\times\vartheta_t^E\right)+\eta_S\cdot\left(\left(1-\mathrm{P}_t^E\right)\times\left(\mho_t^2\right)^E\times\vartheta_t^E\right)+\epsilon_t. \tag{10}$$

Notice that this model implicitly assumes that attitudes toward ambiguity are linearly contingent upon the expected probability of favorable returns $1-\mathrm{P}_t^E$, where P_t^E is the expected probability of unfavorable returns for time t, estimated from historical data.

To interpret the discrete model, using the theoretical model in Equation (5), we can write the coefficient of ambiguity attitude as $\eta\left(1-\mathrm{P}_i^E\right)=\hat{\eta}+\hat{\eta}_i$. This expression then carries the meaning of the investor's attitude toward ambiguity, conditional upon the expected probability of favorable returns $1-\mathrm{P}^E$ being in probability bin i. A negative value of $\eta\left(1-\mathrm{P}_i^E\right)$, determined by $\hat{\eta}+\hat{\eta}_i$, implies ambiguity-loving behavior and results in a negative ambiguity premium. On the other hand, a positive value implies aversion to ambiguity and results in a positive ambiguity premium. Furthermore, a greater $|\hat{\eta}_i|$ in the range of low probabilities of favorable returns implies an increasing love for ambiguity. On the other hand, a higher $\hat{\eta}_i$ in the range of high probabilities of favorable returns implies an increasing aversion to ambiguity. Notice that, since probabilities are additive, a higher probability of favorable returns implies a lower probability of unfavorable returns.

4.2 Empirical findings

The regression model designed in the previous section can now be estimated. The dependent variable in all the regression tests is the monthly excess return on the SPDR. We first examine the model using ordinary least squares (OLS). We use the Newey-West estimator to deal with potential autocorrelation and heteroskedasticity in the error terms of the model. Panel A of Table 4 reports the empirical findings of these regressions.

[INSERT TABLE 4]

The first regression examines the risk–return relationship excluding ambiguity. In this univariate regression, the only explanatory variable is the expected volatility, proxy for risk, which turns out to have a negative coefficient, a result consistent with previous studies; see, for example, French *et al.* (1987). The subsequent regressions introduce expected ambiguity. It can be observed from Panel A that all ambiguity coefficients are significant at the 5% level. Panel B of Table 4 provides the estimates of the level of aversion to or love for ambiguity, contingent upon the expected probability of favorable returns. These estimates are computed for each probability bin i by the expression $\eta\left(1-\mathrm{P}_i^E\right)=\hat{\eta}+\hat{\eta}_i$. For example, Figure 3 depicts the probabilistically contingent coefficients of ambiguity attitude in the third regression, which includes both ambiguity and volatility.

[INSERT FIGURE 3]

To interpret these findings, recall that η_1 is associated with probabilities of favorable returns (gains) in the range $[0.42, 0.436]$, η_2 with probabilities in the range $[0.436, 0.452]$, etc. The evidence is that the coefficient of attitude toward ambiguity associated with probabilities of gains *greater* than 0.532 (i.e., probabilities of losses lower than 0.468) is positive. On the other hand, this coefficient is negative for probabilities of gains *lower* than 0.532. The trend line in Figure 3 shows that ambiguity-averse behavior is demonstrated for probabilities of *favorable* returns greater than 0.5, while ambiguity-loving behavior is demonstrated for probabilities of *unfavorable* returns lower than 0.5. This means that, typically, investors exhibit aversion to ambiguity when they expect gains, while they exhibit love for ambiguity when they expect losses. These findings support Hypotheses 2 and 3, and are consistent with the findings of previous behavioral studies; see, for example, Mangelsdorff and Weber (1994), Abdellaoui *et al.* (2005), and Du and Budescu (2005). However, unlike these studies, in our model investors' preferences for ambiguity are contingent upon the probability of returns rather than upon the returns themselves. It is important to note that we do not discuss our results with respect to other empirical findings about ambiguity attitudes, since we are not aware of any other empirical study of attitudes toward ambiguity that uses trading data.

The decrease in the coefficient of ambiguity attitude from its highest value of 1 to its lowest value of -2.5 indicates that aversion to ambiguity decreases with the expected probability of unfavorable returns and turns to love for ambiguity when this probability exceeds 0.5. This implies that investors' attitudes toward ambiguity are not of the constant absolute class.[28] The behavioral literature has not yet provided conclusive evidence about whether investors' attitudes toward ambiguity are of the constant relative class or of the constant absolute class.[29] To test whether investors' attitudes toward ambiguity are of the latter class, using the same regression format as in Equation (9), we replace the independent variables representing expected ambiguity with $\frac{\left(\mho_t^2\right)^E}{1-\mathrm{P}_t^E}$, i.e., the expected ambiguity normalized by the expected probability of favorable returns. The coefficients obtained in these tests (not reported) show the same pattern as in the regression with the non-normalized ambiguity, implying that investors' attitudes toward ambiguity are not of the constant relative class either. The conclusion is, consistent with Hypothesis 4, that investors exhibit increasing (relative) aversion to ambiguity in the probability of favorable returns and increasing (relative) love for ambiguity in the probability of unfavorable returns.

Hypothesis 4 and the related findings imply an inverse S-shape in perceived probabilities. These findings coincide with Abdellaoui *et al.* (2011), who attribute this shape to different sources of uncertainty, which convert those subjective probabilities into the willingness to bet. They show that these probabilities depend "not only on the person but also on the source of uncertainty. More importantly, our findings support the idea of likelihood insensitivity proposed by Abdellaoui *et al.* (2011). *Likelihood insensitivity* means "a lack of sensitivity to intermediate changes in likelihood, so

[28] Constant absolute ambiguity attitude means that, given a return, when the range of its possible probabilities is shifted linearly, the attitude toward ambiguity remains unchanged.

[29] Constant relative ambiguity attitude means that, given a return, when the range of its possible probabilities is shifted linearly, the attitude toward ambiguity changes proportionally to the magnitude of the shift in probabilities.

that all intermediate likelihoods are moved in the direction of 50-50," suggesting that "decisions will not be influenced much by the updating of probabilities after receipt of new information." In line with the likelihood insensitivity phenomenon, our findings indicate that, for the intermediate range of probabilities, investors do not exhibit a clear sensitivity to ambiguity.

In line with French *et al.* (1987), the effect of expected volatility on expected return in the univariate OLS regressions is generally negative and significant. As can be observed in Panel A of Table 4, when ambiguity is introduced along with volatility, the latter becomes positive and very close to being significant, which can be interpreted as a possible explanation for the equity premium puzzle. The R^2 in Panel A indicates that expected ambiguity is the main explanatory variable. Though the results of the OLS tests regarding ambiguity are encouraging, to further investigate the effect of risk on expected return we repeat our tests using weighted least squares (WLS) regressions, where the weights are inversely proportional to the estimated variance of the residuals. Table 5 reports the empirical findings of these regressions.

[INSERT TABLE 5]

It can be observed from the first regression in Panel A of Table 5 that the effect of expected volatility, by itself, is negative and significant. As in the OLS regressions, the effect of expected ambiguity is significant in all but one of the ten probability bins; see the second regression in Panel A. When expected ambiguity is added to expected volatility, the effect of expected volatility (risk) becomes positive and significant, while the effect of expected ambiguity remains almost the same as in the previous regression. In these regression tests, the intercept is positive and significant, indicating that some of the variation in expected return remains unexplained and there may be other unidentified factors. The coefficient of expected volatility is 1.67. The coefficients of expected ambiguity range from -2.93 to 3.49. The findings of the WLS regressions seem to support our main model, Equation (5), which contains two factors: expected risk and expected ambiguity.

Another interesting finding is with regard to risk attitudes. The coefficient of relative risk aversion can be elicited using the values of the coefficients obtained in Table 5. The obtained value, $2 \times \hat{\gamma} = 3.34$, is in the range of values that have been reported in many behavioral studies (e.g., Chetty (2006) and Booij and Praag (2009)) and empirical asset pricing studies (e.g., Bliss and Panigirtzoglou (2004)). This level of aversion to risk also coincides with the estimates obtained in Brown and Gibbons (1985) and French *et al.* (1987).

Though the empirical findings are in line with previous behavioral finding, our main concern is that these results may be affected by the division of the probability space into 10 probability bins. Although the pattern of the coefficient of ambiguity attitude, $\eta\left(1-\mathrm{P}^{E}\right)$, does not indicate a clear linear relation between attitude toward ambiguity and the expected probabilities of favorable returns, this relation can be viewed as roughly positively monotone. Therefore, we examine a continuous (with respect to ambiguity attitude) version of the model, formed by Equation (10).

As in our previous tests, we test the model using both OLS and WLS regressions. Panel A of Table 6 presents the results of the former and Panel B those of the latter. One can see that the results of these regression tests are quite similar to those of the discrete model with 10 probability bins. Expected volatility turns out to be positive and significant, in the WLS regressions, when expected ambiguity is included. The coefficient of expected volatility, however, is 7.21, more than twice the coefficient in the discrete model, indicating a higher aversion to risk. The reason is that this model is less precise, in the sense that it imposes a linear ambiguity attitude. The coefficients of expected ambiguity in both regression tests are significant and indicate an increasing aversion to ambiguity in the probability of favorable returns. Examining the OLS regression with both expected volatility and expected ambiguity, we observe that investors exhibit aversion to ambiguity when the probability of favorable returns $1-\mathrm{P}^{E} > \frac{5.23}{9.97} = 0.55$. Otherwise, when $1-\mathrm{P}^{E} < 0.55$, they exhibit a love for ambiguity. The explanatory power of the continuous model is lower than that of the discrete model—$R^2 = 0.147$.

[INSERT TABLE 6]

5 Robustness tests

The empirical findings are encouraging, consistent with previous behavioral studies, and support our theoretical model and the related hypotheses. However, we are still concerned that our findings may be due to our methodology or to other, unaccounted for, risk factors. We next try to rule out some alternative explanations for our results.

5.1 VIX as expected volatility

In the previous tests we have estimated expected volatility using essentially historical observations. An alternative approach is to use forward-looking volatility, such as the volatility implied by options on the S&P500 index, given by VIX. We use the VIX value, at the beginning of month t, as a measure of the expected volatility for that month. We then estimate the following model:

$$r_t - r_{f,t} \quad = \quad \alpha + \gamma \cdot VIX_t + \eta \cdot \left(\left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \eta_S \cdot \left(\left(1 - \mathrm{P}_t^E \right) \times \left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \epsilon_t \qquad (11)$$

Table 7 reports the results of the empirical model specified in Equation (11). In both OLS and WLS tests, the findings regarding ambiguity are almost identical to the earlier findings from testing Equation (10). In particular, the coefficients of ambiguity are of the same magnitude and show the same pattern—increasing in the probabilities of favorable returns. We also test the discrete model of Equation (9), where VIX_t is taken to be the expected volatility (not reported). Again, the coefficients of ambiguity are of the same magnitude and show the same pattern as in the discrete model using estimated expected volatility. These findings alleviate our concern that the previous results could have been due to a latent relationship between the estimates of expected volatility and expected ambiguity.

The third WLS regression in Table 7 shows that the relationship between expected ambiguity and expected return is very similar to that obtained in Table 6 using the estimated expected volatility. The relationship between expected volatility, measured by VIX, and expected return is also maintained. It is positive (2.71) and significant (5.34). Using VIX, the coefficient of relative risk aversion is 5.42, while using ARMA the value is 7.21. Since the coefficient obtained using VIX is closer to the behavioural studies' evidence, these findings indicate that VIX may be a better proxy for expected volatility.

[INSERT TABLE 7]

5.2 Is ambiguity a proxy for other factors?

The methodology used to estimate ambiguity raises the question of whether it is just a proxy for other known factors that may affect the return (e.g., skewness, kurtosis, volatility of volatility, etc.). To address this concern, we perform a couple of tests.

We first examine the correlations between expected volatility, expected ambiguity and the expectations of other moments of the empirical distribution. Panel A of Table 8 depicts the correlations among all the variables used in the robustness tests. We start by considering the third and fourth moments, i.e., the *excess skewness* and *excess kurtosis*, computed from the daily returns of the SPDR and adjusted for dividends, as additional factors.[30] Their expected values are estimated using the same methodology as was used for all other variables. These two moments should not be related to ambiguity but they could possibly be related to volatility. The reason is that volatility, skewness and kurtosis are stake-dependent measures, i.e., as functions of returns, they change with the magnitude

[30] We also test the model using skewness, and kurtosis computed from intraday data, taking the expected values of their monthly averages as additional factors.

of the return, while ambiguity is stake-independent as it is solely a function of probabilities. Indeed, as can be observed in Panel A, the correlations of both expected skewness and expected kurtosis, with expected ambiguity (respectively, -8.3% and 13.4%), are insignificant.

[INSERT TABLE 8]

We next introduce into the continuous model of Equation (10) other factors that may affect the expected return, alongside expected ambiguity. The same tests are repeated for the discrete model (not reported), using Equation (9), where for both models we run both OLS and WLS regressions. The regression results in Panel B show that the two factors, expected skewness and expected kurtosis, introduced alongside expected volatility and expected ambiguity, have no effect on expected return. Their coefficients are negligible and in most cases insignificant. Furthermore, introducing these factors alongside expected ambiguity does not affect the significance of the latter. These findings strengthen our claim that the estimated probabilities are not derived from sampling the same (skewed or leptokurtic) probability distribution over the month.

Another concern regarding our methodology for estimating ambiguity is that the measure $\mho^2$ may be an outcome of *time-varying risk* or *volatility innovation*; see, for example, Brandt and Kang (2004). To address this concern, we test our model by introducing expected *volatility of volatility* (VolVol) into the regression, alongside expected volatility and expected ambiguity. VolVol is the variance of the daily variances, computed from intraday data, and its expectation is estimated similarly to those of the other factors. VolVol can be thought of as an estimate of changes in volatility over time. Panel A in Table 8 shows that expected ambiguity and expected VolVol are correlated at 41%, which is significant. Expected VolVol is also correlated with expected volatility, at 64% and also significant. Panel B shows that the inclusion of expected VolVol itself is significant but it has a negligible effect on the coefficients of expected volatility and expected ambiguity, and on their significance. Therefore, we can rule out the possibility that the results are driven by the volatility of volatility.

Since VolMean, the variance of the daily means, has been used as a proxy for ambiguity in some studies (see, for example, Maccheroni *et al.* (2013)) we examine the possibility that ambiguity is derived from expected VolMean. As can be seen in Panel A in Table 8, expected ambiguity and expected VolMean are uncorrelated. Though expected VolMean and expected volatility are correlated, the effect of the former on the coefficients of expected ambiguity and on their significance is negligible. The appendix provides a theoretical discussion of the limitations of using VolVol and VolMean as measures of ambiguity.

Another argument that can be made is that all probabilities within a month are derived from the same probability distribution, which does not vary over the month. To address this concern we conduct the following simulation. Assuming that monthly returns are determined by a single probability distribution, we compute the mean and variance using all intraday returns during the month. We then randomly form (without repetition) 22 groups of 78 return observations each. For each group we compute its mean and variance. Based on these 22 mean-variance pairs, we compute the probabilities and the ambiguity for each month and rerun the regression given by Equation (9). We repeat this procedure 1,000 times, and generally the simulations do not present the pattern of ambiguity attitude obtained from a straightforward use of the data as dictated by our empirical design. Furthermore, in most cases the results of these simulations are not statistically significant.

Next, we use a nonparametric test to test the hypothesis that all daily return distributions over a month are identical and are the same as the monthly return distribution. For each month, we conduct two-sample Kolmogorov-Smirnov tests among all pairs of 20-22 daily probability distributions, where the null hypothesis is that, in every pair, the two distributions are identical. In total, we conduct 53,790 tests and in 51,937 of these tests the null hypothesis is rejected at the 5% level. Second, for each month, we conduct one-sample Kolmogorov-Smirnov test between each daily probability distribution, and a reference probability distribution defined by all intraday observations during the month. In total, we conduct 5,320 tests and in 5,134 of these tests the null hypothesis is rejected at the 5% level. These findings rule out the concern that, contrary to the ambiguity approach, there is a single unique prior rather than a set of priors.

Other robustness tests that we conduct include the following. We relax the assumption that intraday returns are normally distributed, and execute the tests assuming an elliptical distribution with different parameters.[31] The results are qualitatively the same. We also examine our model when downside risk is included as an additional factor (Ang *et al.* (2006)).[32] We conduct these tests with different cutoffs for downside risk, and the results are qualitatively the same. In addition, we test for the effect of investors' sentiment (Baker and Wurgler (2006)) alongside ambiguity, and again the results are qualitatively the same.

Other concern is that our results are driven by the nature of the SPDR, which we take as a

[31] Elliptical distributions are uniquely determined by the mean and variance and can be skewed and at the same time platykurtic or leptokurtic. Particular forms of the elliptical distribution include the normal distribution, student-t distribution, logistic distribution, exponential power distribution and Laplace distribution; see, for example, Owen and Rabinovitch (1983).

[32] Our measure of ambiguity is different than downside risk. While downside risk is stake-dependent, measured by the variance of returns that are lower than a given threshold, ambiguity is stake-independent, measured by the variance of probabilities, independent of returns.

proxy for the market portfolio. Therefore, we also test our model using another proxy for the market portfolio—a value-weighted portfolio of all the stocks quoted in the TAQ database—and repeat the entire estimation process. The results are qualitatively the same as the results obtained using the SPDR.

Our last concern regarding the methodology for estimating ambiguity is that the measure $\mho^2$ might only capture variations in a singleton information set (a single probability distribution) across trading days over a month. That is to say, variations in the returns' distribution across days are derived only from new information that might be obtained overnight. To address this concern, we run all the tests omitting all trading transactions that occur during the first half-hour of the trading day. Berry and Howe (1994) measure the flow of public information to financial markets and show that this flow peaks between 4:30 and 5:00 P.M., after the market has closed. This implies that the most significant impact of the information flow (on bid-ask spread, volatility and trading volume) occurs in the first half-hour of the following trading day. In addition, it has been shown that "price discovery" (price formation) happens mainly in this part of the trading day; see, for example, Lockwood and Linn (1990), Heston *et al.* (2010) and Pagano *et al.* (2013). Omitting the transactions in the first half-hour of the trading day eliminates the effect that overnight information might have. The results of the tests excluding these transactions are essentially the same, ruling out the possibility that our findings are due only to changes in a single information set.

To rule out the possibility that the measure $\mho^2$ only captures the changes in a single probability distribution caused by news clustered on Mondays and Fridays, we run all the tests omitting these two days. The return on the market portfolio might be affected by macroeconomic news that are announced on Mondays and Fridays. Chang *et al.* (1998) and Steeley (2001), for example, show that the main effect of macroeconomic news occurs on these days, since most of the macroeconomic are clustered on these two days (for example the unemployment rate and nonfarm payroll are always announced on Fridays) . It has also been shown that firm-specific announcements are typically clustered on weekends such that their main effect is on Mondays; see, for example, Dyl and Maberly (1988), Schatzberg and Datta (1992), and Damodaran (1989).[33] As before, the results of our tests, when excluding Mondays and Fridays, are essentially the same.

Overall, our basic results withstood all the robustness tests we conducted. We could find no evidence that our ambiguity measure is a proxy for some other known factors.

[33] For a detail survey on the "Monday effect", see Pettengill (2003).

6 Conclusion

The basic tenet in asset pricing is the risk–return relationship, which has been tested a multitude of times using a variety of models and factors. The results have been mixed at best, raising a major concern regarding this most fundamental relationship in finance. One possibility is that ambiguity is an important missing factor that can restore the risk–return relationship by expanding it to include ambiguity.

The current study introduces an empirical measure of ambiguity, based upon a decision theoretic model. It uses this measure in conjunction with measures of risk in time-series tests. This study shows that excess return on the market as a whole, known as the equity premium, is determined by two orthogonal factors: ambiguity and risk. Risk is measured in a variety of ways, e.g., using a rate of return volatility or alternatively implied volatility. Ambiguity is measured by the volatility of the probabilities of returns. To this end, this paper introduces an empirical methodology for measuring ambiguity and for eliciting probabilistically contingent preferences directly from trading data.

Our main hypothesis is that both risk and ambiguity affect the excess return. Consistent with the asset pricing paradigm of risk aversion, one would expect excess return and risk to be positively and linearly related, as in a world of constant relative risk aversion investors. Regarding ambiguity, however, we have no a priori view about its effect on expected excess return. Though many behavioral experiments document an aversion to ambiguity when it comes to gains and a love for ambiguity when it comes to losses, the particular form of the attitude toward ambiguity has yet to be determined. Our findings are rather encouraging. In the case of a high probability of losses, the effect of ambiguity is negative and highly significant, while in the case of a high probability of gains it is positive and highly significant. Furthermore, our findings indicate that aversion to ambiguity increases with the expected probability of gains, while love for ambiguity increases with the expected probability of losses. When we introduce ambiguity into the pricing model, the effect of risk is positive and significant, while its effect is insignificant when ambiguity is not accounted for. The positive equity premium now contains not only a premium for risk but a premium for ambiguity too.

The empirical findings provide a possible explanation for the equity premium puzzle. The empirical methodology proposed in this paper for estimating ambiguity from the data can be employed in other economic and financial studies and might shed light on some anomalies that previously could not be fully explained.

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A Appendix

Why not use only VolVol or only VolMean as a measure of ambiguity?

The variance of variance and the variance of the mean are sometimes interpreted as measures of ambiguity (see, for example, Bloom (2009) and Maccheroni *et al.* (2013)) while assuming either a known mean or a known variance. The measure of ambiguity $\mho^2$, introduced in Izhakian (2014b), encompasses both the variance of variance and the variance of the mean, as well as the variances of the higher moments of the probability distribution (i.e., skewness, kurtosis, etc.), through the variance of probabilities.[34]

There are serious issues with using either the variance of variance on its own or the variance of the mean on its own as a measure of ambiguity. The measure $\mho^2$ presented here does not have these issues. To illustrate this, consider the following two assets: asset A with returns (in percentage points) $r = (-1, 0, 1)$ and, respectively, probabilities $\mathrm{P} = (0.5, 0, 0.5)$, and asset B with the same returns but with two equally likely possible probability distributions $\mathrm{P}_1 = (0.4, 0.2, 0.4)$ and $\mathrm{P}_2 = (0.3, 0.4, 0.3)$. The expected return of asset A is $\mu^A = 0$, and the expected return of asset B is either $\mu_1^B = 0$ or $\mu_2^B = 0$, conditional upon the realized probability distribution. Measuring ambiguity by the variance of the mean implies that both A and B have zero ambiguity. However, by definition (and as $\mho^2$ indicates), B is more ambiguous than A.

Considering the variance of variance as a measure, take the following example. Take asset A with returns $r = (-1, 0, 1)$ and respective probabilities $\mathrm{P} = (0.48, 0.04, 0.48)$, and asset B with the same returns but with two equally likely possible probability distributions $\mathrm{P}_1 = (0.6, 0, 0.4)$ and $\mathrm{P}_2 = (0.4, 0, 0.6)$. The variance of the returns of asset A is $\left(\sigma^A\right)^2 = 0.96$, and the variance of the returns of asset B is either $\left(\sigma_1^B\right)^2 = 0.96$ or $\left(\sigma_2^B\right)^2 = 0.96$, conditional upon the realized probability distribution. Using the variance of variance as a measure of ambiguity implies that both A and B have zero ambiguity. However, by definition, B is more ambiguous than A.

Both the variance of variance and the variance of the mean are functions of returns, which makes them stake-dependent. The advantage of $\mho^2$ is that, unlike these measures, it is stake-independent. It is important to note that the measure of ambiguity $\mho^2$ is not affected by the magnitude of returns, in general, or by the magnitude of unfavorable or favorable returns, in particular. Increasing or decreasing the return of an asset does not change its degree of ambiguity, but it does change its degree of risk.

[34] The importance of unknown variance is expressed, for example, in Epstein and Ji (2013). In empirical asset pricing and macroeconomic contexts, stochastic time-varying volatility also plays an important role; see, for example, Bollerslev *et al.* (1988), Fernández-Villaverde *et al.* (2010) and Bollerslev *et al.* (2011).

B Tables and figures

Figure 1: **Time series of market ambiguity and excess return**
This figure presents the realized market ambiguity and excess return for the period between February 1993 and December 2013. The SPDR is taken as a proxy for the market. The x-axis shows the timeline. The y-axis of the upper plot shows the monthly level of ambiguity $\mho$, measured by the standard deviation of the daily probabilities of returns over the month. Probabilities of returns are computed based upon the daily mean and variance of returns on the SPDR, computed from five-minute returns, taken from the TAQ database and normalized to daily terms. The lower plot depicts the monthly, adjusted for dividends, excess return on the SPDR. The blue dotted vertical lines denote special events that had a significant impact on monthly excess returns.

Figure 2: **Time series of the volatility**
This figure illustrates the realized volatility between February 1993 and December 2013. The x-axis shows the timeline. The y-axis shows the volatility in terms of standard deviation. The volatility, ν, is the variance of the daily returns, adjusted for dividends and normalized to monthly terms, of the SPDR over a month.

Figure 3: **Ambiguity attitudes**
This figure depicts the ambiguity attitude contingent upon the probability of favorable returns. The x-axis shows the probability of favorable returns. The y-axis shows the coefficient of the ambiguity attitude. The solid broken line depicts the ambiguity attitude, and the smooth line is created by a polynomial of third degree. The coefficient of the ambiguity attitude, η, contingent upon the probability of favorable returns over the month, is obtained by regressing the excess return on expected ambiguity and expected volatility. The monthly probabilities of favorable returns are the average daily probabilities of favorable returns over the month. The probabilities of favorable returns are computed based upon the daily mean and variance of returns on the SPDR, computed from five-minute returns, taken from the TAQ database and normalized to daily terms. A return is considered unfavorable if it is lower than the risk-free rate, where returns are assumed to be normally distributed. A negative value of η implies ambiguity-loving behavior and a positive value implies ambiguity-averse behavior.

Table 1: **Summary statistics**

Descriptive statistics are reported for the sample of returns between February 1993 and December 2013.
Panel A reports the summary statistics of the daily parameters employed to compute probabilities of returns. N is the number of five-minute returns in different five-minute time intervals. The mean return, μ, is the daily average five-minute SPDR (Ticker: SPY) return, normalized to daily terms, where intraday returns are computed using prices taken from the TAQ database. σ is the daily standard deviation of five-minute returns, normalized to daily terms. Probabilities of unfavorable returns, P, are computed based upon the daily mean, μ, and variance, σ^2, of the return, assuming a normally distributed return. A return is considered unfavorable if it is lower than the risk-free rate.
Panel B reports the summary statistics of the dependent and main independent variables related to pricing. The monthly risk-free rate of return, r_f, is the one-month Treasury bill rate of return, taken from Ibbotson Associates. The market return, r, is the monthly return, adjusted for dividends, of the exchange-traded fund SPDR, taken from the CRSP database. The volatility, $\sqrt{\nu}$, is the standard deviation of the daily return, adjusted for dividends and normalized to monthly terms. The absolute deviation ϑ is the average absolute daily deviation of returns from the monthly average daily return. The mean probability, $\overline{\mathrm{P}}$, is the average daily probability of unfavorable returns over a month. The ambiguity, $\mho$, is the standard deviation of the daily probabilities of returns over the month.

		N	Mean	Median	Min	Max	Std. Dev	Skewness	Kurtosis
				Panel A: daily descriptive statistics					
n	Num. of obs.	5266	69.791	78.000	3.000	78.000	17.854	-2.024	2.533
μ	Mean return	5266	0.000	0.000	-0.083	0.090	0.010	-0.051	8.125
σ	Std. dev.	5266	0.008	0.007	0.000	0.074	0.005	3.196	18.636
μ/σ	Mean/Std	5266	0.056	0.047	-10.088	9.517	1.153	0.088	2.818
P	Probability	5266	0.492	0.489	0.000	1.000	0.304	0.027	-1.260
				Panel B: monthly descriptive statistics					
r_f	Risk-free rate	251	0.002	0.003	0.000	0.006	0.002	-0.050	-1.523
r	Return	251	0.008	0.013	-0.165	0.109	0.043	-0.688	1.169
$r-r_f$	Excess return	251	0.006	0.010	-0.166	0.109	0.043	-0.671	1.155
$\sqrt{\nu}$	Volatility	251	0.052	0.046	0.007	0.280	0.032	2.884	15.114
VIX	VIX	251	0.060	0.055	0.030	0.175	0.024	1.668	4.310
ϑ	Avg. absolute dev.	251	0.044	0.038	0.013	0.222	0.027	2.722	12.268
$\overline{\mathrm{P}}$	Mean Prob.	251	0.492	0.495	0.294	0.687	0.065	-0.079	0.009
$\mho$	Ambiguity	251	0.378	0.361	0.158	1.303	0.133	1.744	8.617

Panel C: cross- correlations

	$r-r_f$	$\sqrt{\nu}$	ϑ	$\overline{\mathrm{P}}$	$\mho$
$r-r_f$	1				
$\sqrt{\nu}$	-0.337	1			
	(<.0001)				
ϑ	-0.379	0.915	1		
	(<.0001)	(<.0001)			
$\overline{\mathrm{P}}$	-0.621	0.192	0.235	1	
	(<.0001)	(.0023)	(.0002)		
$\mho$	-0.033	-0.103	-0.159	-0.185	1
	(.6034)	(.1036)	(.0116)	(.0033)	

p-values appear in parenthesis.

Table 2: **Autocorrelations**

Autocorrelations are reported for the monthly observations between February 1993 and December 2013, 251 months in total. The autocorrelations are reported for the monthly parameters employed in the empirical tests. The volatility, ν, is the variance of the daily returns, adjusted for dividends and normalized to monthly terms, of the SPDR over a month. The absolute deviation ϑ is the average absolute daily deviation of returns from the monthly average daily return. The mean probability, $\overline{\mathrm{P}}$, is the average daily probability of unfavorable returns over a month. A return is considered unfavorable if it is lower than the risk-free rate, where returns are assumed to be normally distributed. Probabilities are computed based upon the daily mean and variance of returns computed from the five-minute returns, taken from the TAQ database and normalized to daily terms. The ambiguity, $\mho^2$, is the variance of the daily probabilities of returns over the month.

		ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	ρ_9	ρ_{10}
ν	Volatility	0.538	0.175	0.125	0.148	0.149	0.087	0.050	0.072	0.085	0.073
		(<.0001)	(.0057)	(.05)	(.02)	(.0191)	(.1761)	(.4342)	(.2615)	(.1854)	(.2617)
$\ln \frac{\nu_t}{\nu_{t-1}}$	Volatility change	-0.374	-0.052	-0.044	-0.028	-0.024	-0.020	0.079	-0.020	0.028	-0.014
		(<.0001)	(.4166)	(.4881)	(.6606)	(.7056)	(.7553)	(.2196)	(.7536)	(.6654)	(.8254)
ϑ	Absolute dev.	0.743	0.581	0.469	0.396	0.344	0.290	0.252	0.238	0.205	0.186
		(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(.0002)	(.0014)	(.0038)
$\ln \frac{\vartheta_t}{\vartheta_{t-1}}$	Absolute dev. change	-0.330	-0.001	-0.093	0.007	-0.058	0.009	-0.005	-0.001	0.082	-0.089
		(<.0001)	(.9918)	(.1461)	(.9145)	(.3629)	(.8906)	(.9346)	(.9891)	(.2052)	(.1681)
$\overline{\mathrm{P}}$	Mean Prob.	0.110	0.131	0.084	-0.005	0.143	0.085	0.111	0.087	0.195	0.003
		(.0831)	(.0384)	(.1879)	(.9431)	(.025)	(.1847)	(.0825)	(.1754)	(.0023)	(.9581)
$\ln \frac{\overline{\mathrm{P}}_t}{\overline{\mathrm{P}}_{t-1}}$	Mean Prob. change	-0.509	0.043	0.015	-0.123	0.111	-0.069	0.064	-0.083	0.153	-0.160
		(<.0001)	(.5018)	(.8203)	(.0539)	(.0831)	(.2819)	(.3243)	(.1972)	(.0175)	(.013)
$\mho^2$	Ambiguity	0.421	0.232	0.103	0.108	0.098	0.048	0.045	0.030	0.067	0.095
		(<.0001)	(.0002)	(.104)	(.0905)	(.1258)	(.4521)	(.4802)	(.6368)	(.3017)	(.1418)
$\ln \frac{\mho^2{}_t}{\mho^2_{t-1}}$	Ambiguity change	-0.334	-0.056	-0.062	0.026	-0.012	-0.046	0.037	-0.076	-0.080	0.147
		(<.0001)	(.3831)	(.3292)	(.6901)	(.8538)	(.4742)	(.5711)	(.2399)	(.2177)	(.0225)

Table 3: **Expected values**

Expected values are reported for the monthly observations between February 1995 and December 2013, 216 months in total.

Panel A reports summary statistics of the estimated expected values of the factors. The expected volatility ν_t^E is estimated by $\nu_t^E = \exp\left(2\widehat{\ln\sqrt{\nu_t}} + 2\mathrm{Var}\left[u_t\right]\right)$, where $\widehat{\ln\sqrt{\nu_t}}$ is computed using the coefficients estimated by

$$\ln\sqrt{\nu_t} = \varphi_0 + \epsilon_t + \sum_{i=1}^{p} \varphi_i \cdot \ln\sqrt{\nu_{t-i}} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{t-i},$$

and $\mathrm{Var}\left[u_t\right]$ is the minimal predicted variance of the error term. The expected absolute deviation ϑ^E is estimated in a similar way. The expected probability of unfavorable returns is estimated by $\mathrm{P}_t^E = \frac{\exp\left(\widehat{\ln Q_t} + \frac{1}{2}\mathrm{Var}[u_t]\right)}{1+\exp\left(\widehat{\ln Q_t} + \frac{1}{2}\mathrm{Var}[u_t]\right)}$, where $Q_t = \frac{\overline{\mathrm{P}}_t}{1-\overline{\mathrm{P}}_t}$ and $\widehat{\ln Q_t}$ is computed using the coefficients of

$$\ln Q_t = \varphi_0 + \epsilon_t + \sum_{i=1}^{p} \varphi_i \cdot \ln Q_{t-i} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{t-i}.$$

The expected ambiguity is estimated by $\left(\mho_t^2\right)^E = \exp\left(2\widehat{\ln\mho_t} + 2\mathrm{Var}\left[u_t\right]\right)$, where $\widehat{\ln\mho_t}$ is computed using the coefficients of

$$\ln\mho_t = \varphi_0 + \epsilon_t + \sum_{i=1}^{p} \varphi_i \cdot \ln\mho_{t-i} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{t-i}.$$

For each month t the expected values were estimated based upon their realized values in the months from month $t-35$ to month t and using the model with the minimal AIC out of the $n \times m = 100$ combinations of the coefficients.

The volatility, ν, is the variance of the daily returns, adjusted for dividends and normalized to monthly terms, of the SPDR over a month. The absolute deviation ϑ is the average absolute daily deviation of returns from the monthly average daily return. The mean probability, $\overline{\mathrm{P}}$, is the average daily probability of unfavorable returns over a month. A return is considered unfavorable if it is lower than the risk-free rate, where returns are assumed to be normally distributed. Probabilities are computed based upon the daily mean and variance of returns computed from the five-minute returns, taken from the TAQ database and normalized to daily terms. Ambiguity, $\mho^2$, is the variance of the daily probabilities of returns over the month.

Panel B reports the cross-correlations of the estimated expected values. These serve as the main independent variables in the empirical pricing tests.

Panel A: descriptive statistics of forecasted variables

	N	Mean	Median	Min	Max	Std. Dev	Skewness	Kurtosis
$\sqrt{\nu^E}$	216	0.053	0.050	0.019	0.261	0.026	3.716	22.532
ϑ^E	216	0.045	0.042	0.016	0.194	0.022	2.720	13.685
VIX	216	0.063	0.060	0.030	0.175	0.024	1.666	4.386
$\overline{\mathrm{P}}^E$	216	0.491	0.491	0.378	0.608	0.042	-0.203	0.042
$\mho^E$	216	0.369	0.339	0.179	0.992	0.110	1.367	4.272

Panel B: cross-correlations of expected values of the variables

	$\sqrt{\nu^E}$	ϑ^E	VIX	$\overline{\mathrm{P}}^E$	$\mho^E$
$\sqrt{\nu^E}$	1				
ϑ^E	0.835	1			
	(<.0001)				
VIX	0.767	0.858	1		
	(<.0001)	(<.0001)			
$\overline{\mathrm{P}}^E$	0.132	0.188	0.139	1	
	(.0529)	(.0057)	(.0415)		
$\mho^E$	-0.064	-0.190	-0.298	-0.221	1
	(.3459)	(.0050)	(<.0001)	(.0011)	

p-values appear in parenthesis.

Table 4: **Main OLS regression tests**

This table reports the empirical results of the tests of the theoretical discrete model proposed in the paper. The estimated expected values are for the period between February 1996 and December 2013, 216 monthly observations in total.
Panel A reports the results obtained using OLS regressions defined by

$$r_t - r_{f,t} = \alpha + \gamma \cdot \nu_t^E + \eta \cdot \left(\left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \sum_{i=2}^{10} \eta_i \cdot \left(D_{i,t} \times \left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \epsilon_t.$$

All results utilize the Newey-West estimator. The estimated expected value of each variable at time t is the fitted value of an ARMA model over its realized values in the months from month $t-35$ to month t. The expected volatility, ν^E, is estimated from the variance of daily SPDR returns, adjusted for dividends and normalized to monthly terms. The expected absolute deviation ϑ^E is estimated from the average absolute daily deviation of returns from the monthly average daily return. The expected ambiguity, $\left(\mho_t^2\right)^E$, is estimated from the realized ambiguity, where ambiguity $\mho^2$, is the variance of the daily probabilities of returns over the month. Probabilities of returns are computed based upon the daily mean and variance of returns computed from five-minute returns, taken from the TAQ database and normalized to daily terms. A return is considered unfavorable if it is lower than the risk-free rate, where returns are assumed to be normally distributed. The expected probability of unfavorable returns, P^E, is estimated from the monthly averages of the daily probabilities of unfavorable returns. All expectations are estimated using the ARMA model with the minimal AIC. The dummy variable D_i is assigned the value one if the expected probability of favorable returns, $1-\mathrm{P}^E$, in that month falls into the range i of probabilities, and otherwise is assigned the value zero.
Panel B reports the estimated coefficient of ambiguity attitude, calculated as $\eta\left(\mathrm{P}_i^E\right) = \hat{\eta}+\hat{\eta}_i$ using the estimated coefficient of the regression tests in Panel A.

Panel A: regression tests

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\eta}$	$\hat{\eta}_2$	$\hat{\eta}_3$	$\hat{\eta}_4$	$\hat{\eta}_5$	$\hat{\eta}_6$	$\hat{\eta}_7$	$\hat{\eta}_8$	$\hat{\eta}_9$	$\hat{\eta}_{10}$	N	R^2	Adj. R^2
1	0.011	-1.522											216	0.038	0.034
	(3.060)	(-2.910)													
2	0.008		-2.500	1.549	1.350	2.417	2.934	2.301	2.242	3.233	2.938	3.293	216	0.304	0.270
	(1.660)		(-3.880)	(2.040)	(2.020)	(3.740)	(4.420)	(3.580)	(3.440)	(4.890)	(4.340)	(4.860)			
3	0.007	0.531	-2.565	1.602	1.369	2.456	2.926	2.324	2.276	3.281	2.986	3.332	216	0.306	0.268
	(1.510)	(1.690)	(-3.930)	(2.100)	(2.050)	(3.780)	(4.400)	(3.610)	(3.480)	(4.920)	(4.380)	(4.900)			

Panel B: coefficient of ambiguity attitude

P^E	0.420-0.436	0.436-0.452	0.452-0.468	0.468-0.484	0.484-0.500	0.500-0.516	0.516-0.532	0.532-0.548	0.548-0.564	0.564-0.580
2	-2.500	-0.951	-1.151	-0.083	0.434	-0.199	-0.258	0.733	0.437	0.793
3	-2.565	-0.963	-1.196	-0.108	0.361	-0.241	-0.289	0.716	0.421	0.767

t-statistics appear in parenthesis.

Table 5: **Main WLS regression tests**

This table reports the empirical results of the tests of the theoretical discrete model proposed in the paper. The estimated expected values are for the period between February 1996 and December 2013, 216 monthly observations in total.
Panel A reports the results obtained using OLS regressions defined by

$$r_t - r_{f,t} \quad = \quad \alpha + \gamma \cdot \nu_t^E + \eta \cdot \left(\left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \sum_{i=2}^{10} \eta_i \cdot \left(D_{i,t} \times \left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \epsilon_t .$$

All results utilize the Newey-West estimator. The estimated expected value of each variable at time t is the fitted value of an ARMA model over its realized values in the months from month $t-35$ to month t. The expected volatility, ν^E, is estimated from the variance of daily SPDR returns, adjusted for dividends and normalized to monthly terms. The expected absolute deviation ϑ^E is estimated from the average absolute daily deviation of returns from the monthly average daily return. The expected ambiguity, $\left(\mho_t^2\right)^E$, is estimated from the realized ambiguity, where ambiguity $\mho^2$ is the variance of the daily probabilities of returns over the month. Probabilities of returns are computed based upon the daily mean and variance of returns computed from five-minute returns, taken from the TAQ database and normalized to daily terms. A return is considered unfavorable if it is lower than the risk-free rate, where returns are assumed to be normally distributed. The expected probability of unfavorable returns, P^E, is estimated from the monthly averages of the daily probabilities of unfavorable returns. All expectations are estimated using the ARMA model with the minimal AIC. The dummy variable D_i is assigned the value one if the expected probability of favorable returns, $1 - \mathrm{P}^E$, in that month falls into the range i of probabilities and otherwise is assigned the value zero.
Panel B reports the estimated coefficient of ambiguity attitude, calculated as $\eta\left(\mathrm{P}_i^E\right) = \hat{\eta} + \hat{\eta}_i$ using the estimated coefficient of the regression tests in Panel A.

Panel A: regression tests

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\eta}$	$\hat{\eta}_2$	$\hat{\eta}_3$	$\hat{\eta}_4$	$\hat{\eta}_5$	$\hat{\eta}_6$	$\hat{\eta}_7$	$\hat{\eta}_8$	$\hat{\eta}_9$	$\hat{\eta}_{10}$	N	R^2	Adj. R^2
1	0.029	-1.583											216	0.031	0.026
	(7.146)	(-1.596)													
2	0.019		-2.708	1.507	1.711	2.588	3.328	2.502	2.501	3.311	3.289	3.377	216	0.265	0.229
	(3.864)		(-3.420)	(1.643)	(2.081)	(3.262)	(4.173)	(3.167)	(3.124)	(4.142)	(4.055)	(4.169)			
3	0.018	1.672	-2.939	1.658	1.815	2.732	3.271	2.593	2.623	3.465	3.487	3.494	216	0.285	0.246
	(3.745)	(2.378)	(-3.641)	(1.780)	(2.177)	(3.390)	(4.046)	(3.234)	(3.228)	(4.269)	(4.246)	(4.255)			

Panel B: coefficient of ambiguity attitude

P^E			0.420-0.436	0.436-0.452	0.452-0.468	0.468-0.484	0.484-0.500	0.500-0.516	0.516-0.532	0.532-0.548	0.548-0.564	0.564-0.580			
2			-2.708	-1.201	-0.997	-0.120	0.620	-0.206	-0.207	0.603	0.581	0.669			
3			-2.939	-1.281	-1.124	-0.207	0.332	-0.346	-0.317	0.525	0.548	0.555			

t-statistics appear in parenthesis.

Table 6: **Continuous ambiguity attitude**

This table reports the empirical results obtained from testing the theoretical continuous model proposed in the paper. The test covers the period between February 1996 and December 2013, 216 monthly observations in total. **Panel A** reports the results obtained using OLS regressions defined by

$$r_t - r_{f,t} = \alpha + \gamma \cdot \nu_t^E + \eta \cdot \left(\left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \eta_S \cdot \left(\left(1 - \mathrm{P}_t^E\right) \times \left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \epsilon_t .$$

All results utilize the Newey-West estimator. The estimated expected value of each variable at time t is the fitted value of an ARMA model over its realized values in the months from month $t-35$ to month t. The expected volatility, ν^E, is estimated from the variance of daily SPDR returns, adjusted for dividends and normalized to monthly terms. The expected absolute deviation ϑ^E is estimated from the average absolute daily deviation of returns from the monthly average daily return. The expected ambiguity, $\left(\mho_t^2\right)^E$, is estimated from the realized ambiguity, where ambiguity $\mho^2$, is the variance of the daily probabilities of returns over the month. Probabilities of returns are computed based upon the daily mean and variance of returns computed from five-minute returns, taken from the TAQ database and normalized to daily terms. A return is considered unfavorable if it is lower than the risk-free rate, where returns are assumed to be normally distributed. The expected probability of unfavorable returns, P^E, is estimated from the monthly averages of the daily probabilities of unfavorable returns. All expectations are estimated using the ARMA model with the minimal AIC.
Panel B reports the results obtained by testing the above model using WLS regressions.

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\eta}$	$\hat{\eta}_S$	N	R^2	Adj. R^2
				Panel A: OLS			
1	0.011	-1.522			216	0.038	0.034
	(3.060)	(-2.910)					
2	0.010		-5.124	9.820	216	0.160	0.152
	(2.980)		(-5.210)	(5.030)			
3	0.010	0.453	-5.234	9.971	216	0.162	0.150
	(2.790)	(0.580)	(-5.220)	(5.050)			
				Panel B: WLS			
1	0.029	-1.583			216	0.031	0.026
	(7.146)	(-1.596)					
2	0.035		-7.472	14.688	216	0.173	0.165
	(7.659)		(-6.577)	(6.479)			
3	0.029	3.607	-6.157	11.494	216	0.211	0.200
	(6.889)	(5.580)	(-5.703)	(5.360)			

t-statistics appear in parenthesis.

Table 7: **VIX regression tests**

This table reports the empirical results obtained from testing the theoretical continuous model proposed in the paper. The test covers the period between February 1996 and December 2013, 216 monthly observations in total.
Panel A reports the results obtained using OLS regressions defined by

$$r_t - r_{f,t} = \alpha + \gamma \cdot VIX_t + \eta \cdot \left(\left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \eta_S \cdot \left(\left(1 - \mathrm{P}_t^E\right) \times \left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \epsilon_t.$$

All results utilize the Newey-West estimator. VIX_t is the value of VIX at the beginning of month t. The estimated expected value of each variable at time t is the fitted value of an ARMA model over its realized values in the months from month $t-35$ to month t. The expected volatility, ν^E, is estimated from the variance of daily SPDR returns, adjusted for dividends and normalized to monthly terms. The expected absolute deviation ϑ^E is estimated from the average absolute daily deviation of returns from the monthly average daily return. The expected ambiguity, $\left(\mho_t^2\right)^E$, is estimated from the realized ambiguity, where ambiguity $\mho^2$ is the variance of the daily probabilities of returns over the month. Probabilities of returns are computed based upon the daily mean and variance of returns computed from five-minute returns, taken from the TAQ database and normalized to daily terms. A return is considered unfavorable if it is lower than the risk-free rate, where returns are assumed to be normally distributed. The expected probability of unfavorable returns, P^E, is estimated from the monthly averages of the daily probabilities of unfavorable returns. All expectations are estimated using the ARMA model with the minimal AIC.
Panel B reports the results obtained by testing the above model using WLS regressions.

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\eta}$	$\hat{\eta}_S$	N	R^2	Adj. R^2
				Panel A: OLS			
1	0.006	-0.156			216	0.000	-0.004
	(1.650)	(-0.230)					
2	0.010		-5.124	9.820	216	0.160	0.152
	(2.980)		(-5.210)	(5.030)			
3	-0.001	3.163	-6.248	11.825	216	0.216	0.205
	(-0.200)	(3.870)	(-6.270)	(6.030)			
				Panel B: WLS			
1	0.026	3.038			216	0.115	0.112
	(7.383)	(5.695)					
2	0.035		-7.472	14.688	216	0.173	0.165
	(7.659)		(-6.577)	(6.479)			
3	0.038	2.707	-6.183	11.482	216	0.336	0.327
	(10.205)	(5.342)	(-9.113)	(8.492)			

t-statistics appear in parenthesis.

Table 8: **Robustness tests**

This table reports the empirical findings of the robustness tests of the theoretical model proposed in the paper. The test covers the period between February 1996 and December 2013, 216 monthly observations in total.
Panel A reports the cross-correlations among the expected values of the uncertainty factors. The estimated expected value of each variable at time t is the fitted value of an ARMA model over its realized values in the months from month $t-35$ to month t. The expected volatility, ν^E, is estimated from the variance of daily SPDR returns, adjusted for dividends and normalized to monthly terms. The expected absolute deviation ϑ^E is estimated from the average absolute daily deviation of returns from the monthly average daily return. The expected ambiguity, $\left(\mho_t^2\right)^E$, is estimated from the realized ambiguity, where ambiguity $\mho^2$ is the variance of the daily probabilities of returns over the month. Probabilities of unfavorable returns are computed based upon the daily mean and variance of returns computed from five-minute returns, taken from the TAQ database and normalized to daily terms. A return is considered unfavorable if it is lower than the risk-free rate, where returns are assumed to be normally distributed. The expected probability of unfavorable returns, P^E, is estimated from monthly averages of daily probabilities of unfavorable returns. The expected skewness $Skew^E$ is estimated from the realized skewness of daily returns over the month. The expected kurtosis $Kurt^E$ is estimated from the realized kurtosis of daily returns over the month. The expected volatility of the mean $VolM^E$ is estimated from the volatility of the mean, computed as the variance of daily mean returns, which are computed from five-minute returns normalized to daily terms. The expected volatility of volatility $VolV^E$ is estimated from the volatility of volatility, computed as the variance of daily returns, which are computed from five-minute returns normalized to daily terms. All expectations are estimated using the ARMA model with the minimal AIC.
Panel B reports the results obtained using OLS regressions defined by

$$\begin{aligned} r_t - r_{f,t} \quad &= \quad \alpha + \gamma \cdot VIX_t + \eta \cdot \left(\left(\mho_t^2\right)^E \times \vartheta_t^E\right) + \eta_S \cdot \left(\left(1-\mathrm{P}_t^E\right) \times \left(\mho_t^2\right)^E \times \vartheta_t^E\right) + \\ &\quad + \beta_1 \cdot Skew^E + \beta_2 \cdot Kurt^E + \beta_3 \cdot VolM^E + \beta_4 \cdot VolV^E + \epsilon_t. \end{aligned}$$

All results utilize the Newey-West estimator.
Panel C reports the results obtained using WLS regressions.

Panel A: cross-correlations

	$r-r_f$	$\sqrt{\nu}^E$	ϑ^E	P^E	$\mho^E$	$Skew^E$	$Kurt^E$	$VolM^E$	$VolV^E$
$r-r_f$	1								
ν^E	-0.208	1							
	(.0021)								
P^E	-0.180	0.835	1						
	(.0078)	(<.0001)							
P^E	-0.447	0.132	0.188	1					
	(<.0001)	(.0529)	(.0057)						
$\mho^E$	-0.054	-0.064	-0.190	-0.221	1				
	(.4297)	(.3459)	(.005)	(.0011)					
$Skew^E$	0.160	0.019	0.016	0.027	-0.083	1			
	(.0189)	(.7829)	(.8132)	(.6970)	(.2223)				
$Kurt^E$	0.036	0.020	-0.026	-0.130	0.134	-0.068	1		
	(.5965)	(.7671)	(.6992)	(.0569)	(.0489)	(.3200)			
$VolM^E$	-0.221	0.793	0.881	0.156	0.034	-0.017	0.019	1	
	(.0011)	(<.0001)	(<.0001)	(.0219)	(.6188)	(.7982)	(.7834)		
$VolV^E$	-0.290	0.639	0.557	0.038	0.411	-0.006	0.037	0.571	1
	(<.0001)	(<.0001)	(<.0001)	(.5787)	(<.0001)	(.9350)	(.5936)	(<.0001)	

p-values appear in parenthesis.

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\eta}$	$\hat{\eta}_S$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	N	R^2	Adj. R^2
					Panel B: OLS regressions						
1	0.010	0.376	-5.258	10.037	0.021				216	0.186	0.171
	(2.950)	(0.480)	(-5.310)	(5.150)	(2.520)						
2	0.009	0.458	-5.201	9.901		0.001			216	0.162	0.146
	(2.550)	(0.580)	(-5.160)	(4.990)		(0.400)					
3	0.008	0.121	-5.503	10.484			24.209		216	0.163	0.147
	(2.240)	(0.130)	(-5.030)	(4.890)			(0.620)				
4	0.001	0.579	-5.217	10.657				-477595	216	0.202	0.187
	(0.210)	(0.750)	(-5.320)	(5.490)				(-3.260)			
					Panel C: WLS regressions						
1	0.030	3.523	-5.482	10.187	0.013				216	0.192	0.176
	(6.986)	(5.529)	(-5.034)	(4.712)	(1.464)						
2	0.031	3.602	-6.243	11.680		-0.006			216	0.235	0.220
	(7.136)	(5.570)	(-5.812)	(5.476)		(-1.900)					
3	0.018	0.215	-7.263	13.192			296.611		216	0.420	0.409
	(4.253)	(0.303)	(-6.398)	(5.926)			(9.413)				
4	0.014	2.093	-5.021	9.850				-336899	216	0.163	0.147
	(3.090)	(2.985)	(-5.068)	(5.035)				(-1.988)			

t-statistics appear in parenthesis.